Notes: Chain Rule (2)

Find the derivative of each:

Example 1: $f(x) = \sqrt{x^2 + 1}$ $= (x^2+1)^{1/2}$

$\frac{1}{\cancel{2}}(x^2+1)^{-1/2} \cdot (\cancel{2}x) = \frac{x}{\sqrt{x^2+1}}$

Example 2:

A. $f(x) = sin(x^2)$

$f'(x) = \cos(x^2) \cdot [2x]$

$= 2x\cos(x^2)$

B. $f(x) = sin^2 x$

$f(x) = (\sin x)^2$

$= 2(\sin x)'(\cos x)$

$= 2\sin x\cos x$

$(\sin x)(\sin x)$

$(\sin x)\cos x + \sin x\cos x$

$2\sin x\cos x$

Example 3: $f(x) = (x^3 - 1)^{100}$

$f'(x) = 100(x^3-1)^{99}(3x^2)$

$= 300x^2(x^3-1)^{99}$

Example 4: $f(x) = \frac{1}{\sqrt{x^2+x+1}}$ $= (x^2+x+1)^{-1/2}$

$= -\frac{1}{2}(x^2+x+1)^{-3/2}(2x+1)$

$= \frac{-(2x+1)}{2(x^2+x+1)^{3/2}}$

Example 5: $f(x) = \left(\frac{x-2}{2x+1}\right)^9$

$$9\left(\frac{x-2}{2x+1}\right)^8 \frac{d}{dx}\left[\frac{x-2}{2x+1}\right]$$

$$9\left(\frac{x-2}{2x+1}\right)^8 \left[\frac{(2x+1)(1)-(x-2)(2)}{(2x+1)^2}\right]$$

$$\frac{9(x-2)^8}{(2x+1)^8}\left[\frac{\cancel{2x}+1-\cancel{2x}+4}{(2x+1)^2}\right] = \frac{9(x-2)^8(5)}{(2x+1)^{10}}$$

$$= \frac{45(x-2)^8}{(2x+1)^{10}}$$

Example 6: $f(x) = (2x+1)^5(x^3-x+1)^4$

$$(2x+1)^5 \frac{d}{dx}\left[(x^3-x+1)^4\right] + (x^3-x+1)^4 \frac{d}{dx}\left[(2x+1)^5\right]$$

$$(2x+1)^5 4(x^3-x+1)^3(3x^2-1) + (x^3-x+1)^4 5(2x+1)^4(2)$$

$$2(2x+1)^4(x^3-x+1)^3\left[2(2x+1)(3x^2-1) + 5(x^3-x+1)\right]$$

$$2(2x+1)^4(x^3-x+1)^3\left[(4x+2)(3x^2-1) + 5x^3-5x+5\right]$$

$$2(2x+1)^4(x^3-x+1)^3\left[\cancel{12x^3}-\cancel{4x}+6x^2-2+\cancel{5x^3}-\cancel{5x}+5\right]$$

$$2(2x+1)^4(x^3-x+1)^3(17x^3+6x^2-9x+3)$$

Example 7: $f(x) = e^{\sin x}$

$$f'(x) = e^{\sin x} \cdot \cos x$$

$$= \cos x \, e^{\sin x}$$

Example 8: Find the equation of the tangent line to the curve at the given point.

$f(x) = x^2e^{-x} \quad \left(1, \frac{1}{e}\right)$

$$f'(x) = x^2 \frac{d}{dx}\left[e^{-x}\right] + e^{-x}\frac{d}{dx}\left[x^2\right]$$

$$= -x^2e^{-x} + e^{-x}(2x)$$

$$= -x^2e^{-x} + e^{-x}(2x)$$

$$f'(1) = -(1)^2e^{-1} + e^{-1}(2)(1)$$

$$= -e^{-1} + 2e^{-1}$$

$$= e^{-1} = \frac{1}{e}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{e} = \frac{1}{e}(x-1)$$

$$y - \cancel{\frac{1}{e}} = \frac{1}{e}x - \frac{1}{e}$$

$$+\cancel{\frac{1}{e}} \qquad +\frac{1}{e}$$

$$y = \frac{1}{e}x$$