Day 6

AP Calculus-AB

Notes: Limits that Involve Infinity (∞)

Remember:

Vertical Asymptotes Set the denominator of a rational function equal to zero and solve for x.

Horizontal Asymptotes To find, you compare the degree in numerator/denominator.

1. $\frac{\text{degree top smaller}}{\text{degree in bottom}}$ then HA: $y=0$

2. $\frac{\text{degree top equal}}{\text{degree in bottom}}$ then HA: $y=$ leading coeff.

3. $\frac{\text{degree top larger}}{\text{degree in bottom}}$ then HA: none

Example(s) 1: Find all the asymptotes of each function

A.) $f(x)=\frac{4x^1+3}{2x^1-6}$

VA: $2x-6=0$, $2x=6$, $x=3$

HA: $y=\frac{4}{2}$, $y=2$

B.) $g(x)=\frac{3x^0}{x-4}$

VA: $x=4$

HA: $y=0$

C.) $h(x)=\frac{x^2-4}{x^2-x-12}$ $\frac{(x-2)(x+2)}{(x+3)(x-4)}$

VA: $x=-3$ and $x=4$

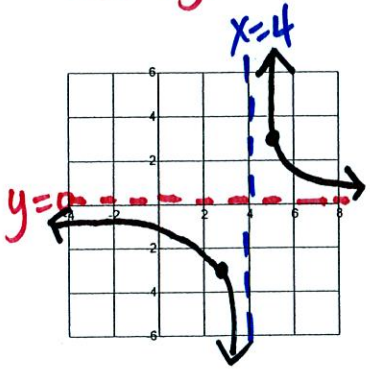
HA: $y=1$

Let look at the graph of B.)

$g(x)=\frac{3}{x-4}$

$g(3)=\frac{3}{3-4}=-3$

$g(5)=\frac{3}{5-4}=3$



How does this apply to calculus?

End Behavior

$\lim_{x\to4^+} g(x)=\infty$

$\lim_{x\to4^-} g(x)=-\infty$

$\lim_{x\to\infty} g(x)=0$

$\lim_{x\to-\infty} g(x)=0$

If $\lim_{x\to c^+} f(x)$ or $\lim_{x\to c^-} f(x)$ Answer $=\pm\infty$ Then VA: $x=c$

If $\lim_{x\to\pm\infty} f(x)=L$ Then HA: $y=L$

How would you answer the same questions without using the graph?

$\lim_{x\to4^+}\frac{3}{x-4}\frac{+}{+}=+\infty$ VA: $x=4$ so Answer $+\infty$ or $-\infty$; $4^+\to4.001$

$\lim_{x\to4^-}\frac{3}{x-4}\frac{+}{-}=-\infty$ VA $x=4$ so Answer $+\infty$ or $-\infty$; $4^-\to3.999$

$\lim_{x\to\infty}\frac{3}{x-4}=0$ EB so Answer HA

$\lim_{x\to-\infty}\frac{3}{x-4}=0$ EB so Answer HA

Example(s) 2: Evaluate each without a calculator:

A.) VA: $x=2$ $\lim_{x\to2^+}\frac{x+3}{x-2}\frac{+}{+}=+\infty$; $2^+\to2.001$

B.) $\lim_{x\to3^+}\ln(x-3)$ $-\infty$

C.) EB $\lim_{x\to\infty}\frac{4x+1}{x-3}=4$ Marilyn

D.) EB $\lim_{t\to-\infty}\frac{t-1}{t^2-4}=0$ J-Lo

E.) EB $\lim_{x\to\infty}\frac{4x^2+1}{x-3}\frac{+}{+}=+\infty$ Dolly goes $\pm\infty$ Big Pos#

F.) EB $\lim_{x\to-\infty}\frac{4x^2+1}{x-3}\frac{+}{-}=-\infty$ Big Neg#

G.) VA: $x=-2$ $\lim_{x\to-2^+}\frac{x}{x+2}\frac{-}{+}=-\infty$; $-2^+\to-1.999$

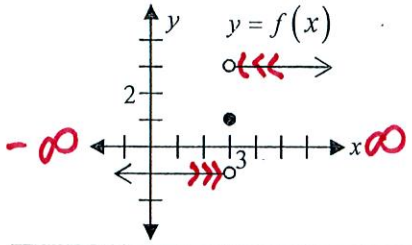
H.) VA: $x=-2$ $\lim_{x\to-2^-}\frac{x}{x+2}=\frac{-}{-}=\infty$; $-2^-\to-2.001$

1. For the function f graphed below, find:

(a) $\lim_{x\to 3^-} f(x)$ -1 (b) $\lim_{x\to 3^+} f(x)$ 3

(c) $\lim_{x\to 3} f(x)$ dne (d) $f(3)$ 1

(e) EB $\lim_{x\to -\infty} f(x)$ -1 (f) EB $\lim_{x\to +\infty} f(x)$ 3

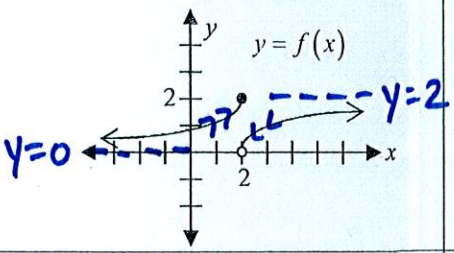


2. For the function f graphed below, find:

(a) $\lim_{x\to 2^-} f(x)$ 2 (b) $\lim_{x\to 2^+} f(x)$ 0

(c) $\lim_{x\to 2} f(x)$ dne (d) $f(2)$ 2

(e) EB $\lim_{x\to -\infty} f(x)$ 0 (f) EB $\lim_{x\to +\infty} f(x)$ 2

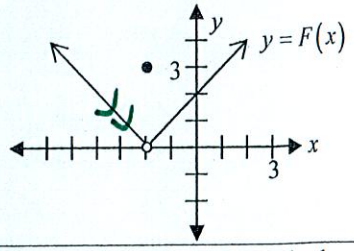


3. For the function F graphed below, find:

(a) $\lim_{x\to -2^-} F(x)$ 0 (b) $\lim_{x\to -2^+} F(x)$ 0

(c) $\lim_{x\to -2} F(x)$ 0 (d) $F(-2)$ 3

(e) $\lim_{x\to -\infty} F(x)$ ∞ (f) $\lim_{x\to +\infty} F(x)$ ∞

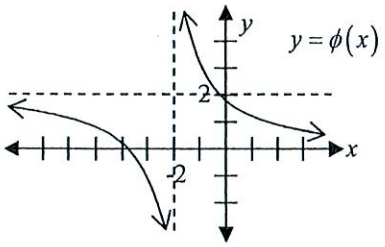


4. For the function ϕ graphed below, find:

(a) $\lim_{x\to -2^-} \phi(x)$ -∞ (b) $\lim_{x\to -2^+} \phi(x)$ ∞

(c) $\lim_{x\to -2} \phi(x)$ dne (d) $\phi(-2)$ undef.

(e) EB $\lim_{x\to -\infty} \phi(x)$ 2 (f) EB $\lim_{x\to +\infty} \phi(x)$ 0

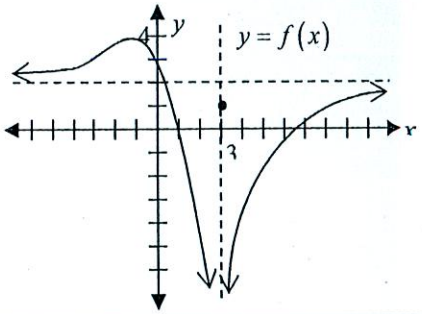


5. For the function f graphed below, find:

(a) $\lim_{x\to 3^-} f(x)$ ∞ (b) $\lim_{x\to 3^+} f(x)$ ∞

(c) $\lim_{x\to 3} f(x)$ ∞ (d) $f(3)$ 1

(e) EB $\lim_{x\to -\infty} f(x)$ 2 (f) EB $\lim_{x\to +\infty} f(x)$ 2

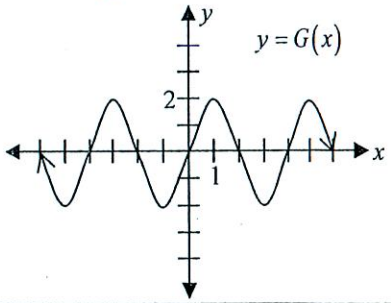


6. For the function G graphed below, find:

(a) $\lim_{x\to 0^-} G(x)$ 0 (b) $\lim_{x\to 0^+} G(x)$ 0

(c) $\lim_{x\to 0} G(x)$ 0 (d) $G(0)$ 0

(e) EB $\lim_{x\to -\infty} G(x)$ d.n.e (f) EB $\lim_{x\to +\infty} G(x)$ d.n.e

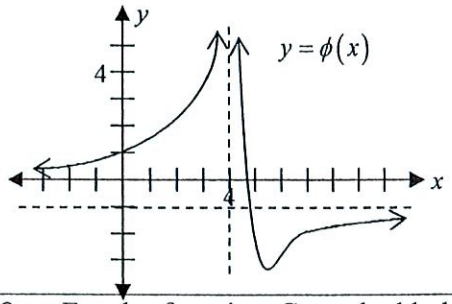


7. For the function ϕ graphed below, find:

(a) $\lim_{x\to 4^-} \phi(x)$ ∞ (b) $\lim_{x\to 4^+} \phi(x)$ ∞

(c) $\lim_{x\to 4} \phi(x)$ ∞ (d) $\phi(4)$ undefined

(e) $\lim_{x\to -\infty} \phi(x)$ 0 (f) $\lim_{x\to +\infty} \phi(x)$ -1

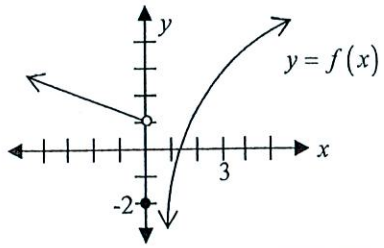


8. For the function f graphed below, find:

(a) $\lim_{x\to 0^-} f(x)$ 1 (b) $\lim_{x\to 0^+} f(x)$ -∞

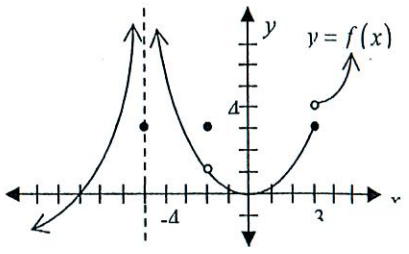
(c) $\lim_{x\to 0} f(x)$ dne (d) $f(0)$ -2

(e) $\lim_{x\to -\infty} f(x)$ ∞ (f) $\lim_{x\to +\infty} f(x)$ ∞



9. Consider the function f, graphed below. For what values of x_0 does $\lim_{x\to x_0} f(x)$ exist?

$(-\infty, 3) \cup (3, \infty)$



8. . For the function G graphed below, find:

(a) $\lim_{x\to 0^-} G(x)$ 3 (b) $\lim_{x\to 0^+} G(x)$ 3

(c) $\lim_{x\to 0} G(x)$ 3 (d) $G(0)$ 3

(e) $\lim_{x\to -\infty} G(x)$ dne (f) $\lim_{x\to +\infty} G(x)$ 0