

Remember:

Vertical Asymptotes: Set the denominator of a rational function equal to zero and solve for x.

Horizontal Asymptotes: To find, you compare the degree in numerator/denominator.

1.  $\frac{\text{degree top smaller}}{\text{degree in bottom}}$

2.  $\frac{\text{degree top equal}}{\text{degree in bottom}}$

3.  $\frac{\text{degree top larger}}{\text{degree in bottom}}$

then

then

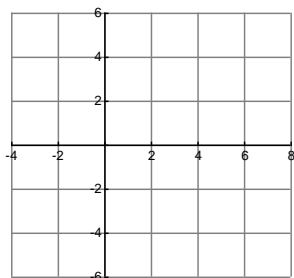
then

**Example(s) 1:** Find all the asymptotes of each function

A.)  $f(x) = \frac{4x+3}{2x-6}$

B.)  $g(x) = \frac{3}{x-4}$

C.)  $h(x) = \frac{x^2-4}{x^2-x-12}$



Let look at the graph of B.)

$g(x) = \frac{3}{x-4}$

How does this apply to calculus?

$\lim_{x \rightarrow 4^+} g(x) = \underline{\hspace{2cm}}$

$\lim_{x \rightarrow 4^-} g(x) = \underline{\hspace{2cm}}$

$\lim_{x \rightarrow \infty} g(x) = \underline{\hspace{2cm}}$

$\lim_{x \rightarrow -\infty} g(x) = \underline{\hspace{2cm}}$

How would you answer the same questions without using the graph?

$\lim_{x \rightarrow 4^+} \frac{3}{x-4} = \underline{\hspace{2cm}}$

$\lim_{x \rightarrow 4^-} \frac{3}{x-4} = \underline{\hspace{2cm}}$

$\lim_{x \rightarrow \infty} \frac{3}{x-4} = \underline{\hspace{2cm}}$

$\lim_{x \rightarrow -\infty} \frac{3}{x-4} = \underline{\hspace{2cm}}$

**Example(s) 2:** Evaluate each without a calculator.

A.)  $\lim_{x \rightarrow 2^+} \frac{x+3}{x-2}$

B.)  $\lim_{x \rightarrow 3^+} \ln(x-3)$

C.)  $\lim_{x \rightarrow \infty} \frac{4x+1}{x-3}$

D.)  $\lim_{t \rightarrow -\infty} \frac{t-1}{t^2-4}$

E.)  $\lim_{x \rightarrow \infty} \frac{4x^2+1}{x-3}$

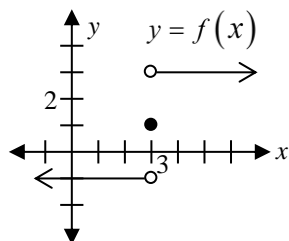
F.)  $\lim_{x \rightarrow -\infty} \frac{4x^2+1}{x-3}$

G.)  $\lim_{x \rightarrow -2^+} \frac{x}{x+2}$

H.)  $\lim_{x \rightarrow -2^-} \frac{x}{x+2}$

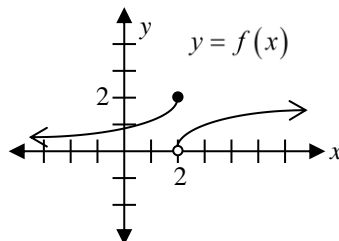
1. For the function  $f$  graphed below, find:

- (a)  $\lim_{x \rightarrow 3^-} f(x)$     (b)  $\lim_{x \rightarrow 3^+} f(x)$   
 (c)  $\lim_{x \rightarrow 3} f(x)$     (d)  $f(3)$   
 (e)  $\lim_{x \rightarrow -\infty} f(x)$     (f)  $\lim_{x \rightarrow +\infty} f(x)$



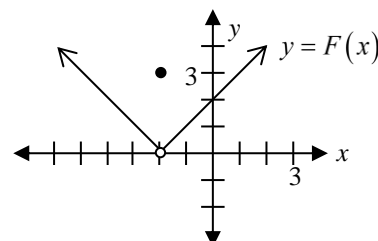
2. For the function  $f$  graphed below, find:

- (a)  $\lim_{x \rightarrow 2^-} f(x)$     (b)  $\lim_{x \rightarrow 2^+} f(x)$   
 (c)  $\lim_{x \rightarrow 2} f(x)$     (d)  $f(2)$   
 (e)  $\lim_{x \rightarrow -\infty} f(x)$     (f)  $\lim_{x \rightarrow +\infty} f(x)$



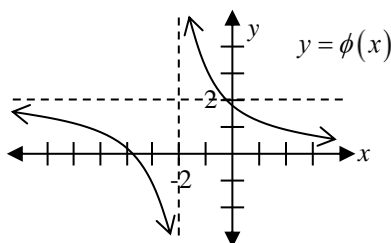
3. For the function  $F$  graphed below, find:

- (a)  $\lim_{x \rightarrow -2^-} F(x)$     (b)  $\lim_{x \rightarrow -2^+} F(x)$   
 (c)  $\lim_{x \rightarrow -2} F(x)$     (d)  $F(-2)$   
 (e)  $\lim_{x \rightarrow -\infty} F(x)$     (f)  $\lim_{x \rightarrow +\infty} F(x)$



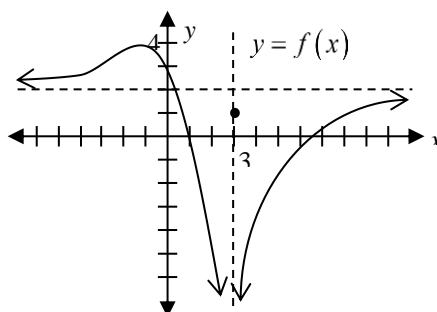
4. For the function  $\phi$  graphed below, find:

- (a)  $\lim_{x \rightarrow -2^-} \phi(x)$     (b)  $\lim_{x \rightarrow -2^+} \phi(x)$   
 (c)  $\lim_{x \rightarrow -2} \phi(x)$     (d)  $\phi(-2)$   
 (e)  $\lim_{x \rightarrow -\infty} \phi(x)$     (f)  $\lim_{x \rightarrow +\infty} \phi(x)$



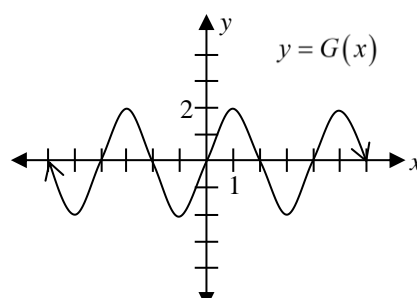
5. For the function  $f$  graphed below, find:

- (a)  $\lim_{x \rightarrow 3^-} f(x)$     (b)  $\lim_{x \rightarrow 3^+} f(x)$   
 (c)  $\lim_{x \rightarrow 3} f(x)$     (d)  $f(3)$   
 (e)  $\lim_{x \rightarrow -\infty} f(x)$     (f)  $\lim_{x \rightarrow +\infty} f(x)$



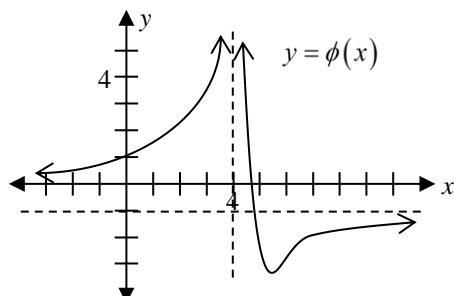
6. For the function  $G$  graphed below, find:

- (a)  $\lim_{x \rightarrow 0^-} G(x)$     (b)  $\lim_{x \rightarrow 0^+} G(x)$   
 (c)  $\lim_{x \rightarrow 0} G(x)$     (d)  $G(0)$   
 (e)  $\lim_{x \rightarrow -\infty} G(x)$     (f)  $\lim_{x \rightarrow +\infty} G(x)$



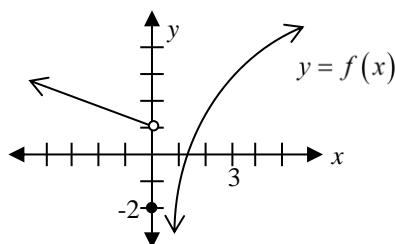
7. For the function  $\phi$  graphed below, find:

- (a)  $\lim_{x \rightarrow 4^-} \phi(x)$     (b)  $\lim_{x \rightarrow 4^+} \phi(x)$   
 (c)  $\lim_{x \rightarrow 4} \phi(x)$     (d)  $\phi(4)$   
 (e)  $\lim_{x \rightarrow -\infty} \phi(x)$     (f)  $\lim_{x \rightarrow +\infty} \phi(x)$

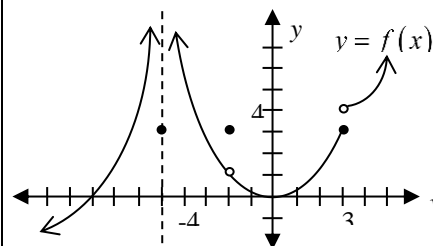


8. For the function  $f$  graphed below, find:

- (a)  $\lim_{x \rightarrow 0^-} f(x)$     (b)  $\lim_{x \rightarrow 0^+} f(x)$   
 (c)  $\lim_{x \rightarrow 0} f(x)$     (d)  $f(0)$   
 (e)  $\lim_{x \rightarrow -\infty} f(x)$     (f)  $\lim_{x \rightarrow +\infty} f(x)$



9. Consider the function  $f$ , graphed below. For what values of  $x_0$  does  $\lim_{x \rightarrow x_0} f(x)$  exist?



8. . For the function  $G$  graphed below, find:

- (a)  $\lim_{x \rightarrow 0^-} G(x)$     (b)  $\lim_{x \rightarrow 0^+} G(x)$   
 (c)  $\lim_{x \rightarrow 0} G(x)$     (d)  $G(0)$   
 (e)  $\lim_{x \rightarrow -\infty} G(x)$     (f)  $\lim_{x \rightarrow +\infty} G(x)$

