

Example One: Evaluate:

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x} = \frac{0}{0} \quad \lim_{x \rightarrow 0} \frac{e^x}{\cos x} = \frac{e^0}{\cos(0)} = \frac{1}{1} = \boxed{1}$$

Example Two:

$$\lim_{x \rightarrow 0} \frac{1}{\sin x} - \frac{1}{x} = \left[\text{Get common denominator} \right] \lim_{x \rightarrow 0} \frac{x \cdot 1}{x \sin x} - \frac{1 \sin x}{x \sin x} = \lim_{x \rightarrow 0} \frac{x - \sin x}{x \sin x} \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \cos x + \sin x(1)} \frac{0}{0} \quad \lim_{x \rightarrow 0} \frac{-(-\sin x)}{x(-\sin x) + \cos x(1) + \cos x} = \frac{0}{0+1+1} = \frac{0}{2} = \boxed{0}$$

Example Three:

$$\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} = \frac{0}{0} \quad \lim_{x \rightarrow 0} \frac{2x}{-(-\sin x)} \frac{0}{0} \quad \lim_{x \rightarrow 0} \frac{2}{\cos x} = \frac{2}{\cos(0)} = \frac{2}{1} = \boxed{2}$$

Example Four:

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = \frac{0}{0} \quad \lim_{x \rightarrow 0} \frac{2e^{2x}}{1} = 2e^0 = 2(1) = \boxed{2}$$

Example Five:

$$\lim_{x \rightarrow \infty} e^{-x} \ln x = \lim_{x \rightarrow \infty} \frac{\ln x}{e^x} \frac{\infty}{\infty} \quad \lim_{x \rightarrow \infty} \frac{\frac{1}{x} \frac{1}{e^x}}{\frac{e^x}{1} \frac{1}{e^x}} = \lim_{x \rightarrow \infty} \frac{1}{x e^x} = \frac{1}{\infty} = \boxed{0}$$

Example Six:

$$\lim_{x \rightarrow \infty} x^3 e^{-x^2} = \lim_{x \rightarrow \infty} \frac{x^3}{e^{x^2}} \frac{\infty}{\infty} \quad \lim_{x \rightarrow \infty} \frac{3x^2}{2x e^{x^2}} = \lim_{x \rightarrow \infty} \frac{3x}{2e^{x^2}} \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow \infty} \frac{3}{2(2x)e^{x^2}} = \boxed{0}$$

Example Seven:

$$\lim_{x \rightarrow 0} x^{\sin x} = 0^0$$

$$y = e^0 = \boxed{1}$$

$$\ln y = \lim_{x \rightarrow 0} \ln x^{\sin x}$$

$$\ln y = \lim_{x \rightarrow 0} \sin x \cdot \ln x$$

$$\ln y = \lim_{x \rightarrow 0} \frac{\ln x}{\csc x}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{1/x}{-\csc x \cot x}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{-1}{x \csc x \cot x}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{-\sin x \tan x}{x} \frac{0}{0}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{-\sin x \sec^2 x + \tan x (-\cos x)}{1} \ln y = \frac{0}{1}$$

$$e^{\ln y} = e^0$$

Example Eight:

$$\lim_{x \rightarrow 0} (1 - 2x)^{\frac{1}{x}} = \ln y = \lim_{x \rightarrow 0} \ln(1 - 2x)^{\frac{1}{x}}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \ln(1 - 2x)$$

$$\ln y = \lim_{x \rightarrow 0} \frac{\ln(1 - 2x)}{x} \frac{0}{0}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{1[-2]}{1 - 2x}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{-2}{1 - 2x}$$

$$\ln y = \frac{-2}{1}$$

$$e^{\ln y} = e^{-2}$$

$$y = e^{-2} = \frac{1}{e^2}$$

Example Nine:

$$\lim_{x \rightarrow 0} (-\ln x)^x = \ln y = \lim_{x \rightarrow 0} \ln(-\ln x)^x$$

$$\ln y = \lim_{x \rightarrow 0} x \cdot \ln(-\ln x)$$

$$\ln y = \lim_{x \rightarrow 0} \frac{\ln(-\ln x)}{x^{-1}}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{1}{\frac{1}{(-\ln x)} \cdot \frac{+1}{x}}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{1}{\frac{1}{x \ln x} \cdot \frac{x^2}{-1}}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{x}{-\ln x}$$

$$\ln y = \lim_{x \rightarrow 0} \frac{1 \cdot \frac{x}{-1}}{\frac{-1}{x} \cdot \frac{x}{-1}}$$

$$\ln y = \lim_{x \rightarrow 0} -x \quad e^{\ln y} = e^0$$

$$y = e^0 = \boxed{1}$$