Day 4

Notes: Chain Rule (1)

■ Chain Rule: is used when you take a derivative of a function "inside" of another function.

$$\frac{d}{dx}[f[g(x)]] =$$

$\frac{d}{dx}[f(g(x))] =$	D18
$\frac{d}{dx}[f(AT)] =$	
$AT = Anything$	

■ outside $(\smile)^2$

■ inside $2x+3$

$f(x) = (2x+3)^2$

Example 1: $f(x) = (2x+3)^2$

A. Find $f'(x)$ the old way.

$f(x) = 4x^2 + 12x + 9$

$f'(x) = 8x + 12$

$f'(x) = 4(2x+3)$

B. Find $f'(x)$ using the Chain Rule

$f'(x) = 2(2x+3)^1 \frac{d}{dx}[2x+3]$

$f'(x) = 2(2x+3)(2)$

$f'(x) = 4(2x+3)$

Example 2: $f(x) = (3x^2 + 4x + 5)^{10}$

Find $f'(x) = 10(3x^2+4x+5)^9 \frac{d}{dx}[3x^2+4x+5]$

$f'(x) = 10(3x^2+4x+5)^9(6x+4)$

Example(s) 3:

A. $f(x) = \sqrt{x}$

$f(x) = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$

B. $f(x) = \sqrt{2x+3}$

$f(x) = (2x+3)^{1/2}$

$f'(x) = \frac{1}{2}(2x+3)^{-1/2} \frac{d}{dx}[2x+3]$

$f'(x) = \frac{1}{\cancel{2}}(2x+3)^{-1/2}(\cancel{2}) = \boxed{\frac{1}{\sqrt{2x+3}}}$

C. $f(x) = \sqrt{sinx}$

$f(x) = (\sin x)^{1/2}$

$f'(x) = \frac{1}{2}(\sin x)^{-1/2} \frac{d}{dx}[\sin x]$

$f'(x) = \frac{1}{2}(\sin x)^{-1/2}\cos x = \boxed{\frac{\cos x}{2\sqrt{\sin x}}}$

D. $f(x) = \sqrt{2xcosx}$

$f(x) = (2x\cos x)^{1/2}$

$f'(x) = \frac{1}{2}(2x\cos x)^{-1/2} \frac{d}{dx}[2x\cos x]$

$f'(x) = \frac{1}{2}(2x\cos x)^{-1/2}\left[2x\frac{d}{dx}[\cos x] + \cos x\frac{d}{dx}[2x]\right]$

$\frac{1}{2\sqrt{2x\cos x}}\left[2x(-\sin x) + \cos x(2)\right]$

$= \frac{\cancel{2}[-x\sin x + \cos x]}{\cancel{2}\sqrt{2x\cos x}} = \boxed{\frac{-x\sin x + \cos x}{\sqrt{2x\cos x}}}$

Day 4 (Continued)

$$\frac{d}{dx}\left[\sec(AT)\right] = \sec(AT)\tan(AT)\frac{d}{dx}[AT]$$

Example(s) 4:

A. $f(x) = secx$

$f'(x) = \sec x \tan x$

B. $f(x) = sec(3x + 4)$

$f'(x) = \sec(3x+4)\tan(3x+4)\frac{d}{dx}[3x+4]$

$f'(x) = 3\sec(3x+4)\tan(3x+4)$

Example 5: $f(x) = \boxed{(3x + 4)^5}\boxed{(2x + 1)^4}$

$f'(x) = (3x+4)^5\frac{d}{dx}\left[(2x+1)^4\right] + (2x+1)^4\frac{d}{dx}\left[(3x+4)^5\right]$

$f'(x) = (3x+4)^5 \cdot 4(2x+1)^3\frac{d}{dx}[2x+1] + (2x+1)^4 \cdot 5(3x+4)^4\frac{d}{dx}[3x+4]$

$f'(x) = (3x+4)^5 \cdot 4(2x+1)^3(2) + (2x+1)^4 \cdot 5(3x+4)^4(3)$

$f'(x) = \underline{8(3x+4)^5(2x+1)^3} + \underline{15(2x+1)^4(3x+4)^4}$

$f'(x) = (3x+4)^4(2x+1)^3\left[\underline{8(3x+4)} + \underline{15(2x+1)}\right]$

$f'(x) = (3x+4)^4(2x+1)^3\left[24x+32+30x+15\right]$

$f'(x) = (3x+4)^4(2x+1)^3(54x+47)$

Example 6: $f(x) = (4x + 5)^{\frac{5}{2}}(2x - 3)^{\frac{7}{2}}$

$f'(x) = (4x+5)^{5/2}\frac{d}{dx}\left[(2x-3)^{7/2}\right] + (2x-3)^{7/2}\frac{d}{dx}\left[(4x+5)^{5/2}\right]$

$f'(x) = (4x+5)^{5/2}\left(\frac{7}{2}\right)(2x-3)^{5/2}\frac{d}{dx}[2x-3] + (2x-3)^{7/2}\left(\frac{5}{2}\right)(4x+5)^{3/2}\frac{d}{dx}[4x+5]$

$f'(x) = (4x+5)^{5/2}\left(\frac{7}{\cancel{2}}\right)(2x-3)^{5/2}(\cancel{2}) + (2x-3)^{7/2}\left(\frac{5}{\cancel{2}}\right)(4x+5)^{3/2}(\cancel{4})2$

$f'(x) = \underline{7(4x+5)^{5/2}(2x-3)^{5/2}} + \underline{10(2x-3)^{7/2}(4x+5)^{3/2}}$

$f'(x) = (4x+5)^{3/2}(2x-3)^{5/2}\left[\underline{7(4x+5)} + \underline{10(2x-3)}\right]$

$f'(x) = (4x+5)^{3/2}(2x-3)^{5/2}\left[28x+35+20x-30\right]$

$f'(x) = (4x+5)^{3/2}(2x-3)^{5/2}\left[48x+5\right]$

Day 4 (Continued)

$$\frac{d}{dx}\left[e^{AT}\right] = e^{AT} \cdot \frac{d}{dx}[AT]$$

Example(s) 7: Find the derivative of each.

A. $f(x) = e^x$

$f'(x) = e^x$

B. $f(x) = e^{3x}$

$f'(x) = e^{3x}\frac{d}{dx}[3x]$

$f'(x) = 3e^{3x}$

C. $f(x) = e^{x^2+4x+5}$

$f'(x) = e^{x^2+4x+5}\frac{d}{dx}[x^2+4x+5]$

$f'(x) = (2x+4)e^{x^2+4x+5}$

D. $f(x) = e^{-x}$

$f'(x) = e^{-x}\frac{d}{dx}[-x]$

$f'(x) = -e^{-x}$

E. $f(x) = \boxed{\sin(2x+4)}\boxed{e^{3x-4}}$

$f'(x) = \sin(2x+4)\frac{d}{dx}\left[e^{3x-4}\right] + e^{3x-4}\frac{d}{dx}\left[\sin(2x+4)\right]$

$f'(x) = \underline{\sin(2x+4) \cdot e^{3x-4} \cdot 3} + \underline{e^{3x-4} \cdot \cos(2x+4) \cdot 2}$

$f'(x) = e^{3x-4}\left[3\sin(2x+4) + 2\cos(2x+4)\right]$