Notes: Volume

L.O.R. (Line of Revolution)

Solid Functions:

- $\int_a^b \text{Area} = \int_a^b \pi R^2$
- $\pi \int_{x_1}^{x_2} (\text{function} - \text{L.O.R.})^2 dx$ If you revolve the function around the x-axis OR a line parallel to x.
- $\pi \int_{y_1}^{y_2} (\text{function} - \text{L.O.R.})^2 dy$ If you revolve the function around the y-axis OR a line parallel to y.

Example 1

Find the volume of the solid bounded by $f(x) = \sqrt{x}$, on the interval $1 \le x \le 4$, and rotated about the x-axis.

$\pi \int_{x_1}^{x_2}$ Solid dx (B.C. Revolved about x-axis)

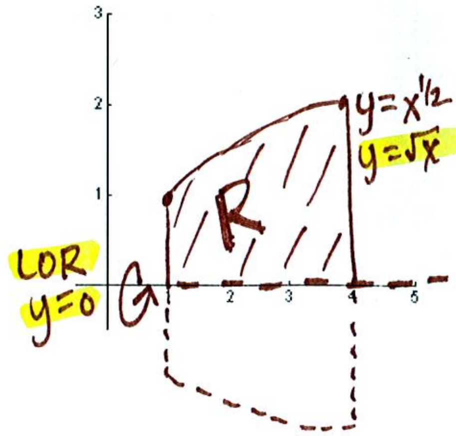
$\pi \int_1^4 (\sqrt{x} - 0)^2 dx$

$\pi \int_1^4 x \, dx$

$\pi \left[\frac{x^2}{2} \right|_1^4$

$\pi \left[\frac{16}{2} - \frac{1}{2} \right]$

$\boxed{\frac{15\pi}{2}}$:) ≈ 23.562



Example 2

$Y_1 = \pi X$
LL:1 UL:4
≈ 23.562

Find the volume of the solid bounded by $f(x) = 2 - x^2$ and the line y=1. That is rotated about the line y=1.

$\pi \int_{-1}^{1} (2 - x^2 - 1)^2 dx$

$\pi \int_{-1}^{1} (-x^2 + 1)^2 dx$

$\pi \int_{-1}^{1} x^4 - 2x^2 + 1 \, dx$

$\pi \left[\frac{x^5}{5} - \frac{2x^3}{3} + x \right|_{-1}^{1}$

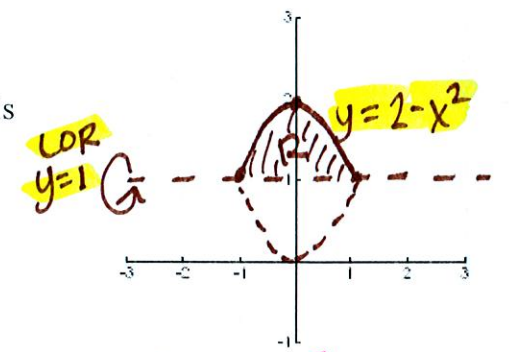
$\pi \left[\left(\frac{1}{5} - \frac{2}{3} + 1 \right) - \left(\frac{-1}{5} + \frac{2}{3} - 1 \right) \right]$

$\pi \left[\frac{1}{5} - \frac{2}{3} + 1 + \frac{1}{5} - \frac{2}{3} + 1 \right.$

$\pi \left[\frac{3 \cdot 2}{3 \cdot 5} - \frac{4 \cdot 5}{3 \cdot 5} + \frac{2 \cdot 15}{15} \right.$

$\pi \left[\frac{6 - 20 + 30}{15} \right] = \boxed{\frac{16\pi}{15}}$:)

≈ 3.351



$Y_1 = \pi (2 - x^2 - 1)^2$
LL:-1 UL:1
$= 3.351$

Example 3

Find the volume of the solid that is bounded by $f(x) = x^3$, y=8, and x=0 that is rotated about the y-axis.

$\sqrt[3]{y} = \sqrt[3]{x^3}$

$x = y^{1/3}$

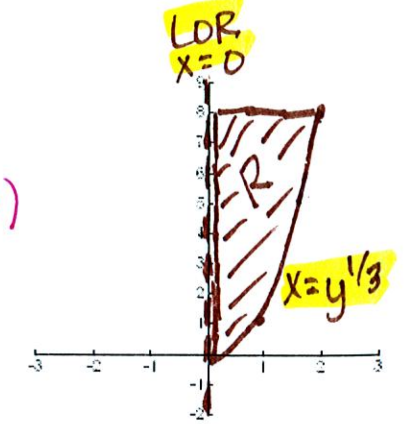
$\pi \int_0^8 (y^{1/3} - 0)^2 dy$

$\pi \int_0^8 y^{2/3} dy$

$\pi \left[\frac{3}{5} y^{5/3} \right|_0^8 = \pi \left[\frac{3}{5} (\sqrt[3]{8})^5 - \frac{3}{5} (\sqrt[3]{0})^5 \right] = \boxed{\frac{96\pi}{5}}$:)

≈ 60.319

$Y_1 = \pi X \wedge (2 \div 3)$
LL:0 UL:8
60.319



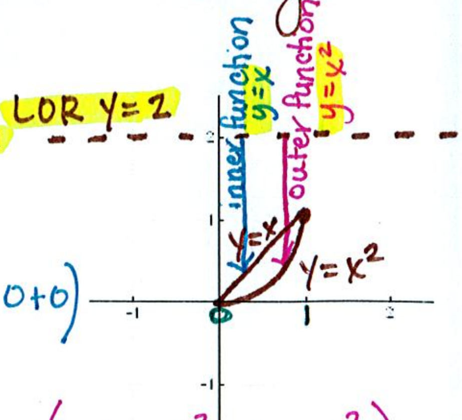
Functions with a Hole

- $\pi \int_{x_1}^{x_2} (\text{outer function} - \text{L.O.R})^2 - (\text{Inner function} - \text{L.O.R.})^2 \, dx$ x or parallel to x
- $\pi \int_{y_1}^{y_2} (\text{outer function} - \text{L.O.R.})^2 - (\text{Inner function} - \text{L.O.R})^2 \, dy$ y or parallel to y

Example 4

Find the volume of the object formed by $f(x) = x^2$ and $y=x$ that is rotated about the line $y=2$.

LOR $y=2$



$\pi \int_0^1 (x^2-2)^2 - (x-2)^2 \, dx$

$\pi \int_0^1 (x^4-4x^2+4) - (x^2-4x+4) \, dx$

$\pi \int_0^1 x^4 - 4x^2 + \cancel{4} - x^2 + 4x - \cancel{4} \, dx$

$\pi \int_0^1 x^4 - 5x^2 + 4x \, dx$

$\pi \left[\frac{x^5}{5} - \frac{5x^3}{3} + \frac{4x^2}{2} \right|_0^1$

$\pi \left[\frac{3 \cdot 1}{3 \cdot 5} - \frac{5 \cdot 5}{3 \cdot 5} + \frac{2 \cdot 15}{15} - (0-0+0) \right.$

$\pi \left[\frac{3-25+30}{15} \right.$

$\boxed{\frac{8\pi}{15}}$:)

≈ 1.676

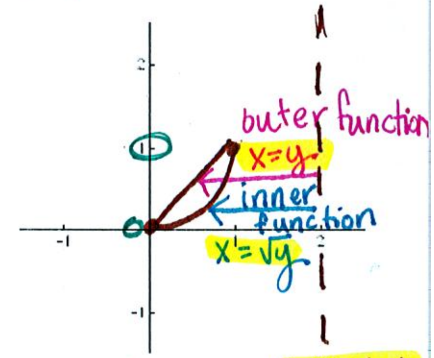
$Y_1 = \pi((x^2-2)^2 - (x-2)^2)$

LL: 0 UL: 1

≈ 1.676

Example 5

Find the volume of the object formed by $f(x) = x^2$ ($x = \sqrt{y}$, $y = x^2$) and $y=x$ that is rotated about the line $x=2$.



$\pi \int_0^1 (y-2)^2 - (\sqrt{y}-2)^2 \, dy$

$\pi \int_0^1 (y^2-4y+4) - (y-4\sqrt{y}+4) \, dy$

$\pi \int_0^1 y^2 - 4y + \cancel{4} - y + 4y^{1/2} - \cancel{4} \, dy$

$\pi \int_0^1 y^2 - 5y + 4y^{1/2} \, dy$

$\pi \left[\frac{y^3}{3} - \frac{5y^2}{2} + \frac{4 \cdot 2}{3} y^{3/2} \right|_0^1$

$\pi \left[\frac{y^3}{3} - \frac{5}{2} y + \frac{8}{3} y^{3/2} \right|_0^1$

$\pi \left[\frac{1}{3} - \frac{5}{2} + \frac{8}{3} - (0-0+0) \right.$

$\pi \left[3 - \frac{5}{2} \right] = \boxed{\frac{\pi}{2}}$:)

≈ 1.571

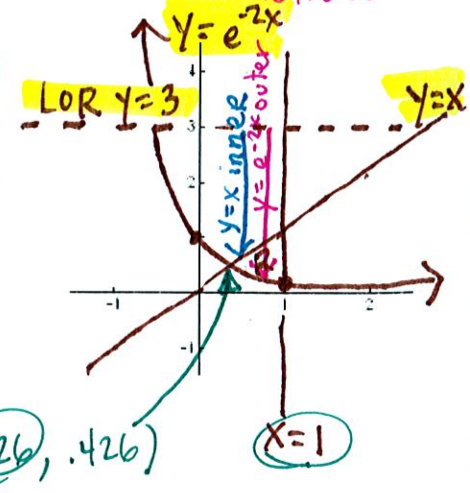
$Y_1 = \pi((x-2)^2 - (\sqrt{x}-2)^2)$

LL: 0 UL: 1

≈ 1.571

Example 6

Find the volume of the object formed by $f(x) = e^{-2x}$, $g(x) = x$ and $x = 1$ that is rotated about the line $y=3$.



$\pi \int_{.426}^1 (e^{-2x}-3)^2 - (x-3)^2 \, dx$

$Y_1 = \pi((e \wedge (-2x) - 3)^2 - (x-3)^2)$

LL: .426

UL: 1