Day 3

Notes: Derivatives Of Trigonometric Functions

$\frac{d}{dx}[sinx] = \underline{\cos x}$ $\qquad$ $\frac{d}{dx}[cosx] = \underline{-\sin x}$

$\frac{d}{dx}[tanx] = \underline{\sec^2 x}$ $\qquad$ $\frac{d}{dx}[cotx] = \underline{-\csc^2 x}$

$\frac{d}{dx}[secx] = \underline{\sec x \tan x}$ $\qquad$ $\frac{d}{dx}[cscx] = \underline{-\csc x \cot x}$

D11	D12	D13
$\frac{d}{dx}[\sin x] =$ $\frac{d}{dx}[\sin AT] =$ $AT = Anything$	$\frac{d}{dx}[\cos x] =$ $\frac{d}{dx}[\cos AT] =$ $AT = Anything$	$\frac{d}{dx}[\tan x] =$ $\frac{d}{dx}[\tan AT] =$ $AT = Anything$

D14	D15	D16
$\frac{d}{dx}[\csc x] =$ $\frac{d}{dx}[\csc x] =$ $AT = Anything$	$\frac{d}{dx}[\sec x] =$ $\frac{d}{dx}[\sec AT] =$ $AT = Anything$	$\frac{d}{dx}[\cot x] =$ $\frac{d}{dx}[\cot AT] =$ $AT = Anything$

Example 1: Prove $\frac{d}{dx}[tanx] = sec^2x$

$$\frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos x \frac{d}{dx}[\sin x] - \sin x \frac{d}{dx}[\cos x]}{[\cos x]^2}$$

$$= \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

Example 2: Prove $\frac{d}{dx}[cscx] = -cscxcotx$

$$\frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{\underbrace{\sin x \frac{d}{dx}[1]}_{0} - (1)\frac{d}{dx}[\sin x]}{[\sin x]^2}$$

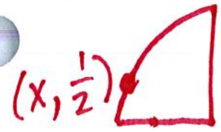
$$= \frac{-1(\cos x)}{\sin x \cdot \sin x} = \boxed{-\csc x \cot x}$$

D3(Cont.)

Example(s) 3:

A. $f(x) = cosx$ find $f'\left(\frac{5\pi}{6}\right) =$ $f'(x) = -\sin x$

$f'\left(\frac{5\pi}{6}\right) = -\sin\left(\frac{5\pi}{6}\right) = \boxed{-\frac{1}{2}}$



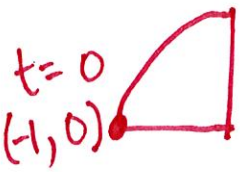
B. $f(x) = tanx$ find $f'\left(\frac{3\pi}{4}\right) =$ $f'(x) = \sec^2 x$

$f'\left(\frac{3\pi}{4}\right) = \left[\sec\frac{3\pi}{4}\right]^2 = \left[-\frac{2}{\sqrt{2}}\right]^2 = \frac{4}{2} = \boxed{2}$

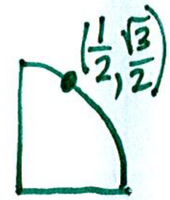
C. $f(x) = secx$ find $f'(\pi) =$ $f'(x) = \sec x \tan x$

$f'(\pi) = \sec\pi \tan\pi$

$\left(\frac{1}{-1}\right)(0) = \boxed{0}$



Example 4: Find the equation of the tangent line for $f(x) = xsinx$ at $x = \frac{\pi}{3}$.



$\boxed{1.}$ Point $\left(\frac{\pi}{3}, \frac{\pi\sqrt{3}}{6}\right)$ $f\left(\frac{\pi}{3}\right) = \frac{\pi}{3}\sin\frac{\pi}{3} = \frac{\pi}{3}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi\sqrt{3}}{6}$

$\boxed{2.}$ Slope: $\frac{\pi}{6} + \frac{\sqrt{3}}{2}$

$f'\left(\frac{\pi}{3}\right)$

$f'(x) = x\frac{d}{dx}[\sin x] + \sin x \frac{d}{dx}[x]$

$f'(x) = x\cos x + \sin x$

$f'\left(\frac{\pi}{3}\right) = \frac{\pi}{3}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}$

$f'\left(\frac{\pi}{3}\right) = \frac{\pi}{3}\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2}$

$y - \frac{\pi\sqrt{3}}{6} = \left(\frac{\pi}{6} + \frac{\sqrt{3}}{2}\right)\left(x - \frac{\pi}{3}\right)$

Example 5: $f(x) = cos^2x$

$f(x) = \cos x \cdot \cos x$ Find $f'(x)$

$f'(x) = \cos x \frac{d}{dx}[\cos x] + \cos x \frac{d}{dx}[\cos x]$

$= -\sin x\cos x - \sin x \cos x$

$= \boxed{-2\sin x\cos x}$

Example 6: $f(x) = sinx(x^2 - 3x)$

Find $f'(x)$

$f'(x) = \sin x \frac{d}{dx}[x^2 - 3x] + (x^2 - 3x)\frac{d}{dx}[\sin x]$

$= \sin x(2x - 3) + (x^2 - 3x)\cos x$

$= \boxed{2x\sin x - 3\sin x + x^2\cos x - 3x\cos x}$

D17

How do you find a horizontal tangent?

Horizontal Tangents: Set $f'(x) = 0$ and solve for x.

Example 7: $f(x) = \sqrt{3}x + 2cosx$ Find the Horizontal Tangent.

$f'(x) = \sqrt{3} + 2(-\sin x)$

Set $f'(x) = 0$ & Solve for x.

$0 = \sqrt{3} - 2\sin x$

$\frac{2\sin x}{2} = \frac{\sqrt{3}}{2}$

$\sin^{-1}(x)$: means find the location on the unit circle that y has that value

domain $(-\pi/2, \pi/2)$

$\cancel{\sin^{-1}\sin} x = \sin^{-1}\frac{\sqrt{3}}{2}$

$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \boxed{\frac{\pi}{3}}$