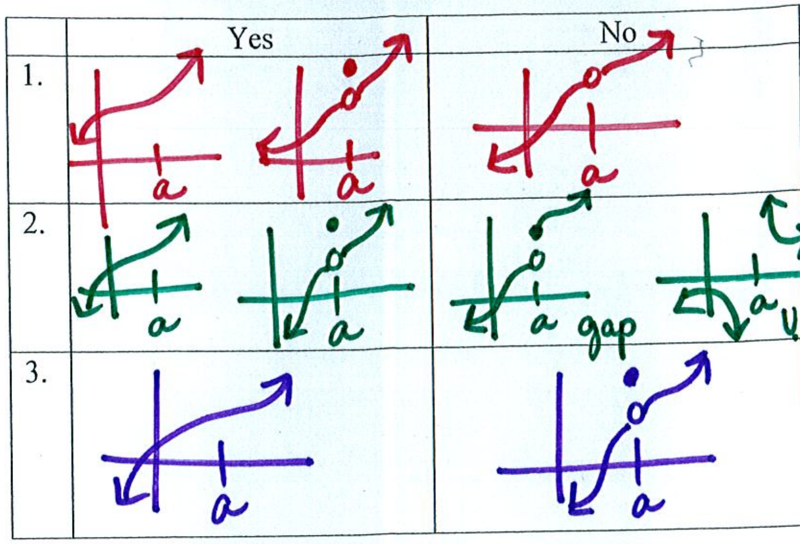
AP Calculus-AB
Notes: Continuity

Continuity: For a function to be continuous at a point 3 conditions must be met at $x = a$.

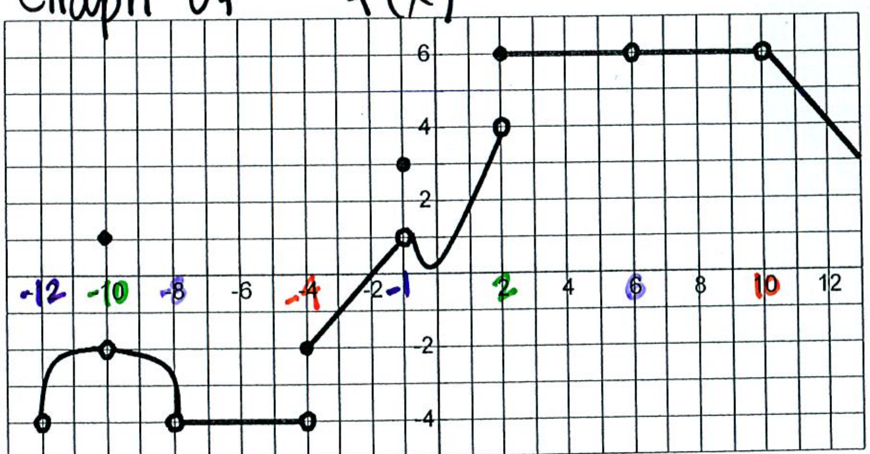
1. $f(a)$ must be defined
2. $\lim\limits_{x\to a} f(x)$ must exist
3. $f(a) = \lim\limits_{x\to a} f(x)$



Example(s) 1:

State all the places the following function is discontinuous and tell why it is discontinuous.

Graph of $f(x)$



Where Discontinuous	Why
$x = -12$	$f(-12)$ is undefined OR $\lim\limits_{x\to 12} f(x)$ does not exist
$x = -10$	$f(-10) \neq \lim\limits_{x\to -10} f(x)$; $1 \neq -2$
$x = -8$	$f(-8)$ is undefined.
$x = -4$	$\lim\limits_{x\to -4} f(x) =$ does not exist
$x = -1$	$\lim\limits_{x\to -1} f(x) \neq f(-1)$; $1 \neq 3$

$x = 2$: $\lim\limits_{x\to 2} f(x) =$ does not exist

$x = 6$: $f(6)$ is undefined

$x = 10$: $f(10)$ is undefined

Example(s) 2:

Determine if each are continuous everywhere. If yes use the 3 step method to prove. If no state where discontinuous and why.

A.) $f(x) = \begin{cases} x^2, & for \quad x > 2 \quad 2^+ \\ 2x, & for \quad x \le 2 \quad 2^- \end{cases}$

$\boxed{1.}$ $f(2) = 2(2) = 4$

$\boxed{2.}$ $\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^-} f(x)$

$(2)^2 = 2(2)$

$4 = 4$

$\boxed{3.}$ $f(2) = \lim\limits_{x\to 2} f(x)$

$4 = 4$

Continuous $(-\infty, \infty)$

B.) $f(x) = \begin{cases} 2x+3, & for \quad x \ge 0 \quad 0^+ \\ -2x-1, & for \quad x < 0 \quad 0^- \end{cases}$

$\boxed{1.}$ $f(0) = 2(0) + 3 = 3$

$\boxed{2.}$ $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^-} f(x)$

$2(0) + 3 = -2(0) - 1$

$3 \neq -1$

Discontinuous at $x = 0$ because $\lim\limits_{x\to 0} f(x)$ does not exist

C.) $f(x) = \begin{cases} x^2+3, & for \quad x > -1 \quad -1^+ \\ 2x+6, & for \quad x < -1 \quad -1^- \\ 2, & for \quad x = -1 \end{cases}$

$\boxed{1.}$ $f(-1) = 2$

$\boxed{2.}$ $\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^-} f(x)$

$(-1)^2 + 3 = 2(-1) + 6$

$4 = 4$

$\boxed{3.}$ $f(-1) \neq \lim\limits_{x\to -1} f(x)$

$2 \neq 4$

Discontinuous at $x = -1$ b.c. $f(-1) \neq \lim\limits_{x\to -1} f(x)$

Removable v/s Non- removable Discontinuities:

→ Removable: Hole

→ Non-removable: A gap OR V.A. (where $\lim_{x \to a} f(x)$ does not exist)

Continuous Functions

Polynomial Functions

→ linear, quadratic, cubic, ...

Absolute Value Functions (that do not have variable in den)

Sinx

cosx

e^x / ln(x) [Exponentials/Logs]

Square Root (on domain)

Non-continuous Functions

Rational Functions

→ Functions w/variable in the bottom

Piecewise (most of them)

Tanx

Cotx

Secx

cscx

Example(s) 3:

Determine if each are continuous everywhere. If yes use the 3 step method to prove. If no state where discontinuous and what kind of discontinuity.

A.) $f(x) = \frac{3}{x+2}$

VA: set bottom of function = 0 and solve for x.

x = -2 is a non-Removable discontinuity.

[B.C. VA x = -2]

B.) $f(x) = \frac{(\cancel{x+3})(x+5)}{(x+2)(\cancel{x+3})}$

hole: If Rational function has a common factor in top/bottom

$\boxed{x=-2}$ is a non-Removable discontinuity asymptote

And

$\boxed{x=-3}$ is a Removable discontinuity hole

C.) $f(x) = \begin{cases} x^3, & for\ x \geq 2 \\ 2x+3, & for\ x < 2 \end{cases}$ 2^+ 2^-

$\boxed{1.}$ $f(2) = 2^3 = 8$

$\boxed{2.}$ $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$

$(2)^3 = 2(2)+3$

$8 \neq 7$

$\boxed{x=2}$ is a non-Removable discontinuity gap