Notes: Fundamental Theorem of Calculus (Part II) Integration Day 5

$$A(x) = \int_a^x f(t)dt = signed\ Area\ from\ a\ to\ x$$

Fundamental Theorem of Calculus (Part II)

Let $f(x)$ be a continuous function on $[a, b]$.

Then $\frac{d}{dx}A(x) = \frac{d}{dx}\int_a^x f(t)\,dt$ is an antiderivative of $f(x)$, that

$A'(x) =$ $f(x)$, or equivalently,

$\frac{d}{dx}\int_a^x f(t)\,dt =$ $f(x)$

What does that mean?

Evaluate: $\cancel{\frac{d}{dx}\int_\pi^x} \boxed{\sin(t)}\,dt =$

$\frac{d}{dx}\left[-\cos t \Big|_\pi^x\right] = \frac{d}{dx}\left[-\cos x - -\cos\pi\right] = \frac{d}{dx}\left[-\cos x - \cancel{1}^{\,0}\right]$

$\boxed{\sin x}$

$- -\sin x = \boxed{\sin x}$

Try Again: $\cancel{\frac{d}{dx}\int_5^x} \boxed{t^2}\,dt =$

$\frac{d}{dx}\left[\frac{t^3}{3}\Big|_5^x\right] = \frac{d}{dx}\left[\frac{x^3}{3} - \frac{5^3}{3}\right] = \frac{d}{dx}\left[\frac{x^3}{3} - \cancel{\frac{125}{3}}^{\,0}\right]$

$\frac{1}{3}x^3$

$\cancel{\frac{1}{3}}(\cancel{3}x^2)$

$\boxed{x^2}$

$= \frac{\cancel{3}x^2}{\cancel{3}} = \boxed{x^2}$

Now what does it mean?

What happens if the upper limit is something other than plain x?

Evaluate: $\cancel{\frac{d}{dx}\int_\pi^{x^2}} \sin(t)\,dt =$

$\frac{d}{dx}\left[-\cos t \Big|_\pi^{x^2}\right] = \frac{d}{dx}\left[-\cos(x^2) - -\cos\pi\right] = \frac{d}{dx}\left[-\cos(x^2) - \cancel{1}^{\,0}\right]$

$= - -\sin(x^2)\cdot 2x = \boxed{2x\sin(x^2)}$

$\boxed{2x\sin(x^2)}$

Try Again: $\cancel{\frac{d}{dx}\int_5^{3x^2}} t^2\,dt =$

$\frac{d}{dx}\left[\frac{t^3}{3}\Big|_5^{3x^2}\right] = \frac{d}{dx}\left[\frac{(3x^2)^3}{3} - \frac{5^3}{3}\right] = \frac{d}{dx}\left[9x^6 - \cancel{\frac{125}{3}}^{\,0}\right]$

$\boxed{54x^5}$

$6x(3x^2)^2$

$6x[9x^4] = \boxed{54x^5}$

Fundamental Thm. of Calculus Part II

Integration Day 5

$\cancel{\frac{d}{dx}\int_{constant}^{f(x)}} f(t)\,dt =$ $f(f(x))\cdot f'(x)$

$$\boxed{\frac{d}{dx}\int_{constant}^{f(x)} f(t)\,dt}$$

Example 1: Evaluate each using the First Fundamental Theorem of Calculus (FFTC)

A. $\frac{d}{dx}\int_0^x \sin^2(t)\,dt =$

$\boxed{\sin^2 x}$

B. $\frac{d}{dx}\int_2^x 3t + \cos(t^2)(t)\,dt =$

$\boxed{3x + \cos(x^2)\cdot x}$

C. $\frac{d}{dx}\int_7^x \frac{1+t}{1+t^2}\,dt =$

$\boxed{\frac{1+x}{1+x^2}}$

D. $\frac{d}{dx}\int_0^{x^2} e^{t^2}\,dt =$

$e^{(x^2)^2}\cdot 2x = \boxed{2xe^{x^4}}$

E. $\frac{d}{dx}\int_6^{x^2} \cot(3t)\,dt =$

$\cot(3x^2)\cdot 2x$

$\boxed{2x\cot(3x^2)}$

F. $\frac{d}{dx}\int_7^{5x} \frac{\sqrt{1+u^2}}{u}\,du =$

$\frac{\sqrt{1+(5x)^2}}{\cancel{5}x}\cdot\cancel{5} = \boxed{\frac{\sqrt{1+25x^2}}{x}}$

G. $\frac{d}{dx}\int_x^6 \ln(1+t^2)\,dt =$

$\frac{d}{dx} - \int_6^x \ln(1+t^2)\,dt =$

$\boxed{-\ln(1+x^2)}$

H. $\frac{d}{dx}\int_{x^3}^5 \frac{cost}{t^2+2}\,dt =$

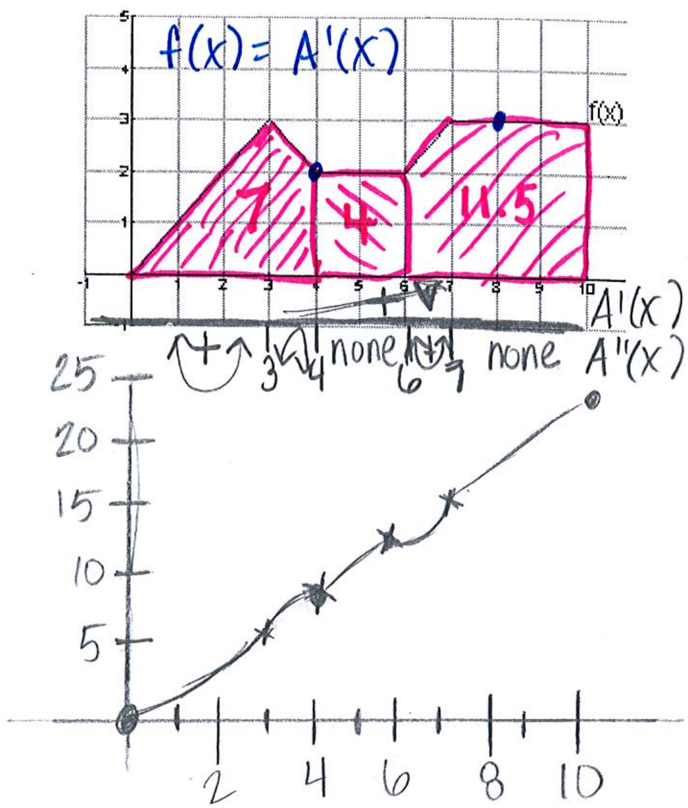
$\frac{d}{dx} - \int_5^{x^3} \frac{\cos t}{t^2+2}\,dt$

$\frac{-\cos(x^3)}{(x^3)^2+2}\cdot 3x^2$

$\boxed{\frac{-3x^2\cos(x^3)}{x^6+2}}$

$A'(x) = f(x)$

Example 2: Let $\frac{d}{dx}A(x) = \frac{d}{dx}\int_0^x f(t)\,dt$



Calculate

A. $A(0) = \int_0^0 f(t)dt = 0$

B. $A(4) = \int_0^4 f(t)dt = 7$

C. $A(6) = \int_0^6 f(t)dt = 7 + 4 = 11$

D. $A(10) = \int_0^{10} f(t) = 11 + 11.5 = 22.5$

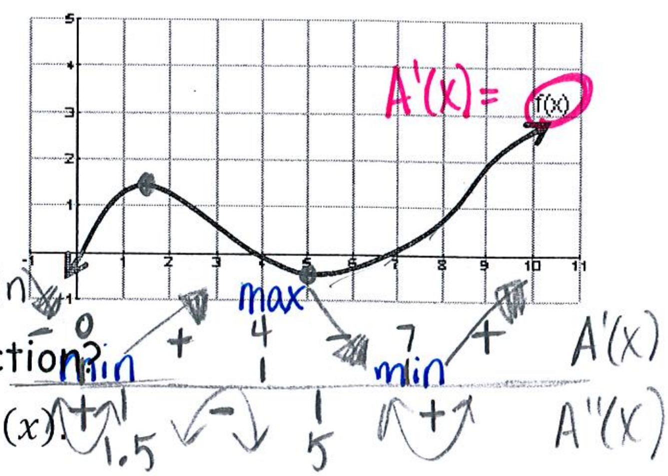
E. $A'(4) = f(4) = 2$

F. $A'(8) = f(8) = 3$

Sketch A(x)

$A'(x) = f(x)$

Example 3: Let $\frac{d}{dx}A(x) = \frac{d}{dx}\int_0^x f(t)\,dt$



A. State the intervals of increasing/decreasing for $A(x)$

increasing: $(0,4) \cup (7,\infty)$

decreasing: $(-\infty,0) \cup (4,7)$

B. Where does $A(x)$ have a local max/min?

C. Where does $A(x)$ have points of inflection?

D. State the intervals of concavity for $A(x)$.

B. max: $x = 4$

min: $x = 0$ & $x = 7$

C. P.O.I.: $x = 1.5$ & $x = 5$

D. Concave up: $(-\infty, 1.5) \cup (5, \infty)$

Concave down: $(1.5, 5)$