$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Notes: *L'Hôpital's Rule*

Example One: Evaluate:

$$\lim_{x\to1}\frac{x^3-1}{x-1} = \frac{0}{0} = \lim_{x\to1}\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}} = \lim_{x\to1} x^2+x+1$$

$$(1)^2+1+1 = \boxed{3}$$

AD 9
When do you use *L'Hôpital's Rule*? How do you use it?

Indeterminate Forms: $\frac{0}{0}, \frac{\infty}{\infty}, \infty-\infty, or\ 0^0$

Use *L'Hôpital's Rule* when you get an Indeterminate Form.

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)} = \lim_{x\to a}\frac{f''(x)}{g''(x)} =$$ You can take a derivative as many times as you need.

- You must have a Fraction to use L'Hopitals Rule.
- You may use as many times as necessary.

Example Two: Rework example one using *L'Hôpital's Rule*

$$\lim_{x\to1}\frac{[x^3-1]\frac{d}{dx}}{[x-1]\frac{d}{dx}} = \frac{0}{0} = \lim_{x\to1}\frac{3x^2}{1} = 3(1)^2 = \boxed{3}$$ ☺

Example Three:

$$\lim_{x\to\frac{\pi}{2}}\frac{cos^2x}{sinx-1} = \frac{0}{0} = \lim_{x\to\frac{\pi}{2}}\frac{[\cos x]^2\frac{d}{dx}}{[\sin x-1]\frac{d}{dx}} = \lim_{x\to\frac{\pi}{2}}\frac{2\cancel{(\cos x)}^1(-\sin x)}{\cancel{\cos x}} = \lim_{x\to\frac{\pi}{2}} -2\sin x$$

$$= -2\sin\left(\frac{\pi}{2}\right) = -2(1) = \boxed{-2}$$

Example Four:

$$\lim_{x\to0^+} x\ln x = \lim_{x\to0^+}\frac{\ln x}{x^{-1}} \cdot \frac{\infty}{\infty} = \lim_{x\to0^+}\frac{\frac{1}{x}}{-x^{-2}} = \lim_{x\to0^+}\frac{\frac{1}{x}\cdot\frac{x^2}{-1}}{\cancel{\frac{-1}{x^2}\cdot\frac{x^2}{-1}}} = \lim_{x\to0^+} -x = \boxed{0}$$

Example Five:

$$\lim_{x\to0}\frac{\sin(4x)}{x^2+3} = \frac{\sin(0)}{0^2+3} = \frac{0}{3} = \boxed{0}$$

Direct substitution works ☺

Remember

$\frac{0}{0}$ = Indeterminant

$\frac{0}{\text{Any \#}} = 0$

$\frac{\text{Any \#}}{0}$ = undefined

Example Six:

$\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x^3-7x-6} = \dfrac{0}{0}$ $\lim_{x\to 3} \dfrac{\left[(x+1)^{1/2}-2\right]\frac{d}{dx}}{\left[x^3-7x-6\right]\frac{d}{dx}} = \lim_{x\to 3} \dfrac{\frac{1}{2}(x+1)^{-1/2}(1)}{3x^2-7}$

$\lim_{x\to 3} \dfrac{1}{2\sqrt{x+1}\,(3x^2-7)} = \dfrac{1}{2(2)(20)} = \boxed{\dfrac{1}{80}}$

$\boxed{\lim_{x\to\infty} \dfrac{\text{Any\#}}{x^p}}$ AD 10

P = positive integer

$= \boxed{0}$

Example Seven:

$\lim_{x\to\infty} \dfrac{\ln x}{\sqrt{x}} = \dfrac{\infty}{\infty}$ $\lim_{x\to\infty} \dfrac{\frac{1}{x}}{\frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \dfrac{\frac{1}{x}}{\frac{1}{2x^{1/2}}} \cdot \dfrac{2x^{1/2}}{1} = \lim_{x\to\infty} \dfrac{2}{x^{1/2}} = \dfrac{2}{\sqrt{\infty}} = \boxed{0}$

Example Eight:

$\lim_{x\to\infty} \dfrac{\ln(x^4+1)}{x} = \dfrac{\infty}{\infty}$ $\lim_{x\to\infty} \dfrac{\frac{1}{x^4+1}(4x^3)}{1} = \lim_{x\to\infty} \dfrac{4x^3}{x^4+1}\ \dfrac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{12x^2}{4x^3} = \lim_{x\to\infty} \dfrac{3}{x} = \boxed{0}$

Example Nine:

$\lim_{x\to\frac{\pi}{2}} \dfrac{\tan(3x)}{\tan(5x)} =$ omit for now :)

Example Ten:

$\lim_{x\to 2} \dfrac{e^{x^2}-e^4}{x-2} = \dfrac{0}{0}$ $\lim_{x\to 2} \dfrac{e^{x^2}(2x)}{1} = 2(2)e^{2^2} = \boxed{4e^4}$

Example Eleven:

$\lim_{x\to\infty} \dfrac{x^2+4x}{9x^3+4x} = \dfrac{\infty}{\infty} = \lim_{x\to\infty} \dfrac{2x+4}{27x^2+4}\ \dfrac{\infty}{\infty} = \lim_{x\to\infty} \dfrac{2}{54x} = \boxed{0}$

$\lim_{x\to\infty} f(x) =$ end behavior so = your ladies so J-Lo so $= \boxed{0}$