Product Rule: If $y = \boxed{f(x)} \cdot \boxed{g(x)}$

- Then $y' = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

  Or

  Mrs. Mac's Version $y' = (first) \cdot \frac{d}{dx}(second) + (second) \cdot \frac{d}{dx}(first)$

| Product Rule | |
|---|---|

- Sometimes you can do Algebra so you don't have to use product rule. Most times if you have a choice, the Algebra is the best way to go.

Example(s) Two: $f(x) = (3x^2 + 4)(2x - 6)$

A. Do some Algebra so you don't have to use product rule

$f(x) = 6x^3 - 18x^2 + 8x - 24$

$f'(x) = 18x^2 - 36x + 8$

B. Use Product Rule

$f(x) = \boxed{(3x^2+4)}\boxed{(2x-6)}$

$f'(x) = (3x^2+4) \cdot \frac{d}{dx}[2x-6] + (2x-6) \cdot \frac{d}{dx}[3x^2+4]$

$f'(x) = (3x^2+4)(2) + (2x-6)(6x)$

$f'(x) = \underline{6x^2} + 8 + \underline{12x^2} - 36x$

$f'(x) = 18x^2 - 36x + 8$

Example(s) Three: $f(x) = \boxed{3x}\boxed{e^x}$

$f'(x) = 3x \cdot \frac{d}{dx}[e^x] + e^x \cdot \frac{d}{dx}[3x]$

$f'(x) = 3x \cdot e^x + e^x(3)$

$f'(x) = 3e^x(x+1)$

You have no choice in this problem. You must use product rule.

$f(x) = 3e^x$ (no product rule)

Quotient Rule: If $y = \dfrac{\boxed{f(x)}\ \text{hi}}{\boxed{g(x)}\ \text{lo}}$

| Quotient Rule | |
|---|---|

Then $$y' = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

Or

Mrs. Mac's Version $$y' = \frac{\boxed{lo}\, d\, \boxed{hi} - \boxed{hi}\, d\, \boxed{lo}}{\boxed{lo}^2}$$

Example(s) Four: $f(x) = \dfrac{\boxed{5x^3 - 4x^2 + 3x - 2}\ \text{hi}}{\boxed{x}\ \text{lo}}$

A. Do some Algebra so you don't have to use the quotient rule

$f(x) = \frac{5x^3}{x} - \frac{4x^2}{x} + \frac{3x}{x} - \frac{2}{x}$

$f(x) = 5x^{2-1} - 4x^{1-1} + \cancel{3} - 2x^{-1-1}$

$f'(x) = 10x - 4 + 2x^{-2}$

$f'(x) = 10x\frac{x^2}{x^2} - 4\frac{x^2}{x^2} + \frac{2}{x^2}$

$f'(x) = \frac{10x^3 - 4x^2 + 2}{x^2}$

B. Use Quotient Rule

$f'(x) = \frac{(x)\cdot\frac{d}{dx}[5x^3 - 4x^2 + 3x - 2] - (5x^3 - 4x^2 + 3x - 2)\frac{d}{dx}[x]}{x^2}$

$f'(x) = \frac{x[15x^2 - 8x + 3] - (5x^3 - 4x^2 + 3x - 2)[1]}{x^2}$

$f'(x) = \frac{15x^3 - \cancel{8x^2} + \cancel{3x} - 5x^3 + \cancel{4x^2} - \cancel{3x} + 2}{x^2}$

$f'(x) = \frac{10x^3 - 4x^2 + 2}{x^2}$

Example(s) Five: $f(x) = \dfrac{3x^2 + 4x}{x+2}$

$f'(x) = \frac{(x+2)\frac{d}{dx}[3x^2 + 4x] - (3x^2 + 4x)\frac{d}{dx}[x+2]}{(x+2)^2}$

$f(x) = \frac{3x^2}{x+2} + \frac{4x}{x+2}$

No help! must use quotient Rule!

$f'(x) = \frac{(x+2)[6x+4] - (3x^2 + 4x)[1]}{(x+2)^2}$

$f'(x) = \frac{\cancel{6x^2} + \cancel{4x} + 12x + 8 - \cancel{3x^2} - \cancel{4x}}{(x+2)^2} = \frac{3x^2 + 12x + 8}{(x+2)^2}$

$24 \cdot \frac{3}{2} = 6$

Example(s) Six: $f(x) = (x^2 - 4x)(x^{\frac{1}{2}} + 2)$

$f(x) = (x^{\frac{4}{2}} - 4x^{\frac{2}{2}})(x^{\frac{1}{2}} + 2)$

$f(x) = x^{\frac{5}{2}} + 2x^2 - 4x^{3/2} - 8x$

$f'(x) = \frac{5}{2}x^{\frac{3}{2}} + 4x - 6x^{1/2} - 8$

Example(s) Seven: $\frac{d}{dx}\left[\frac{e^x}{x^2+1}\right]\bigg|_{x=0}$ *and* $\frac{d}{dx}\left[\frac{e^x}{x^2+1}\right]\bigg|_{x=1}$

$f'(x) = \dfrac{(x^2+1)\cdot\frac{d}{dx}[e^x] - e^x \cdot \frac{d}{dx}[x^2+1]}{(x^2+1)^2}$

$f'(x) = \dfrac{(x^2+1)e^x - e^x(2x)}{(x^2+1)^2}$

$f'(x) = \dfrac{e^x(x^2-2x+1)}{(x^2+1)^2}$

$f'(0) = \dfrac{e^0(0^2-2(0)+1)}{(0^2+1)^2} = 1$

$f'(1) = \dfrac{e^1(1^2-2(1)+1)}{(1^2+1)^2} = \dfrac{e^1(0)}{4} = 0$

Example Eight: Find the tangent line to $f(x) = \dfrac{2x}{x-4}$ at $x = 6$

Tangent line

$\boxed{1.}$ Point $f(6) = \underline{6}$

$\boxed{2.}$ Slope $f'(6) = \underline{-2}$

$f(x) = \dfrac{2x}{x-4}$

$f(6) = \dfrac{2(6)}{6-4} = 6$

$y - 6 = -2(x-6)$

$f'(x) = \dfrac{(x-4)\cdot\frac{d}{dx}[2x] - 2x\cdot\frac{d}{dx}[x-4]}{(x-4)^2}$

$f'(x) = \dfrac{(x-4)(2) - 2x(1)}{(x-4)^2}$

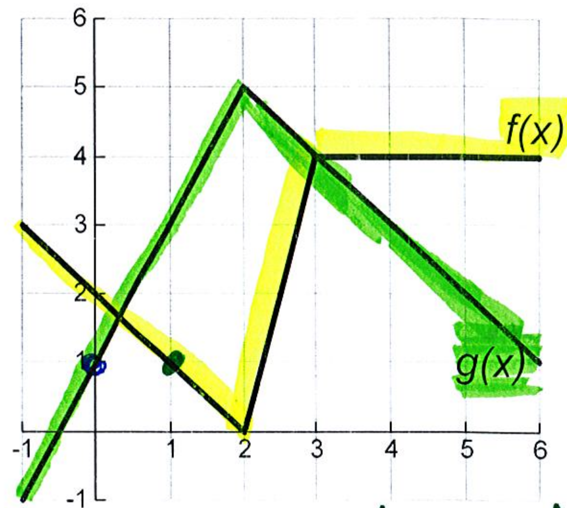
$f'(x) = \dfrac{\cancel{2x} - 8 - \cancel{2x}}{(x-4)^2}$

$f'(x) = \dfrac{-8}{(x-4)^2}$

$f'(6) = \dfrac{-8}{(6-4)^2} = \dfrac{-8}{4} = -2$

Example Nine: If $f$ and $g$ are the functions whose graphs are shown, let

$m(x) = f(x)g(x)$ and $q(x) = \frac{f(x)}{g(x)}$.




A.)Find $m'(1)$

$m(x) = f(x) \cdot g(x)$

$m'(x) = f(x) \cdot g'(x) + g(x)f'(x)$

$m'(1) = f(1) \cdot g'(1) + g(1) \cdot f'(1)$

$m'(1) = 1(2) + 3(-1) = -1$

B.)Find $q'(0)$

$q(x) = \frac{f(x)}{g(x)}$

$q'(x) = \frac{g(x) \cdot f'(x) - f(x)g'(x)}{[g(x)]^2}$

$q'(0) = \frac{g(0) \cdot f'(0) - f(0) \cdot g'(0)}{[g(0)]^2}$

$q'(0) = \frac{(1)(-1) - (2)(2)}{[1]^2}$

$q'(0) = \frac{-5}{1} = -5$

$f(1) =$ what is the y-value on f when x=1?

$f(1) = 1$

$f'(1) =$ what is the slope on f when x=1?

$f'(1) = -1$

$g(1) = 3$

$g'(1) = 2$

$g(0) = 1$

$g'(0) = 2$

$f(0) = 2$

$f'(0) = (-1)$