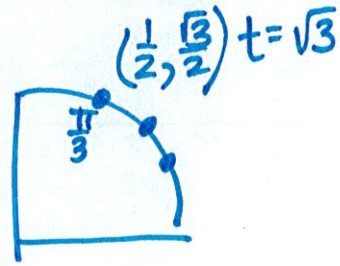
Notes: day 4

AP Calculus-AB

Notes: Finding a Limit Algebraically



3 Ways to Evaluate Limits:

1. Graphically
2. Numerically
3. Algebraically

Algebraically

You should always try to find a limit algebraically first.

1. Plug in the x –value first. If you get an answer that is your limit.
2. If you get an indeterminate form you can try Algebra ☺

    Factor (to see if you can simplify)

    Multiply by the conjugate

    Multiply and simplify

    Get common denominator

Example(s) 1: Just plug in your x-value

A.) $\lim\limits_{x\to 2} x^2+3x+4 = (2)^2+3(2)+4 = 4+6+4 = \boxed{14}$

B.) $\lim\limits_{x\to \frac{\pi}{3}} tanx = \tan\frac{\pi}{3} = \boxed{\sqrt{3}}$

C.) $\lim\limits_{x\to 1} \frac{\sqrt{x+8}}{x-2} = \frac{\sqrt{1+8}}{1-2} = \frac{\sqrt{9}}{-1} = \frac{3}{-1} = \boxed{-3}$

Example(s) 2: (Get zero in bottom) Try factoring

A.) $\lim\limits_{x\to -2} \frac{x^2-4x-12}{x+2}$ $\lim\limits_{x\to -2} \frac{\cancel{(x+2)}(x-6)}{\cancel{x+2}} = \lim\limits_{x\to -2} x-6 = -2-6 = \boxed{-8}$

B.) $\lim\limits_{x\to 3} \frac{x^3-27}{x-3}$ $(x)^3-(3)^3$ $\lim\limits_{x\to 3} \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{x-3}} = \lim\limits_{x\to 3} x^2+3x+9 = (3)^2+3(3)+9 = \boxed{27}$

C.) $\lim\limits_{x\to 4} \frac{\sqrt{x}-2}{x-4}$ $\lim\limits_{x\to 4} \frac{\cancel{\sqrt{x}-2}}{(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}} = \lim\limits_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

$\lim\limits_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)}$ $\lim\limits_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} = \lim\limits_{x\to 4} \frac{1}{\sqrt{x}+2} = \boxed{\frac{1}{4}}$

Example(s) 3: Rationalize Numerator/Denominator

A.) $\lim\limits_{h\to 5}\frac{(h-5)(\sqrt{h+4}+3)}{(\sqrt{h+4}-3)(\sqrt{h+4}+3)}$ $\lim\limits_{h\to 5}\frac{(h-5)(\sqrt{h+4}+3)}{h+4-9} = \lim\limits_{h\to 5}\frac{\cancel{(h-5)}(\sqrt{h+4}+3)}{\cancel{h-5}}$

$\lim\limits_{h\to 5}\sqrt{h+4}+3 = \sqrt{5+4}+3 = 3+3 = \boxed{6}$

B.) $\lim\limits_{x\to 0}\frac{(\sqrt{x+5}-\sqrt{5})(\sqrt{x+5}+\sqrt{5})}{x(\sqrt{x+5}+\sqrt{5})} = \lim\limits_{x\to 0}\frac{x+\cancel{5-5}}{x(\sqrt{x+5}+\sqrt{5})} = \lim\limits_{x\to 0}\frac{\cancel{x}}{\cancel{x}(\sqrt{x+5}+\sqrt{5})}$

$= \lim\limits_{x\to 0}\frac{1}{\sqrt{x+5}+\sqrt{5}} = \frac{1}{\sqrt{0+5}+\sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}$

Example(s) 4: Mult & Collect like terms

A.) $\lim\limits_{h\to 0}\frac{(3a+h)^2-9a^2}{h} = \lim\limits_{h\to 0}\frac{\cancel{9a^2}+6ah+h^2-\cancel{9a^2}}{h} = \lim\limits_{h\to 0}\frac{\cancel{h}(6a+h)}{\cancel{h}}$

$\lim\limits_{h\to 0} 6a+h = 6a+0 = \boxed{6a}$

B.) $\lim\limits_{h\to 0}\frac{2(x+h)^2-2x^2}{h} = \lim\limits_{h\to 0}\frac{2(x^2+2xh+h^2)-2x^2}{h} = \lim\limits_{h\to 0}\frac{\cancel{2x^2}+4xh+2h^2-\cancel{2x^2}}{h}$

$\lim\limits_{h\to 0}\frac{\cancel{h}(4x+2h)}{\cancel{h}} = 4x+2(0) = \boxed{4x}$

Example(s) 5: Get common denominator & Reduce to one fraction

A.) $\lim\limits_{h\to 0}\frac{3\left(\frac{1}{3+h}\right)3 - \frac{1}{3}(3+h)}{h}\;\frac{(3+h)}{} = \lim\limits_{h\to 0}\frac{\frac{\cancel{3-3}-h}{3(3+h)}}{h} = \lim\limits_{h\to 0}\frac{-h}{3(3+h)}\cdot\frac{1}{h}$

$\frac{h}{1} \times \frac{1}{h}$

$\lim\limits_{h\to 0}\frac{-1}{3(3+h)} = \frac{-1}{3(3+0)} = \boxed{\frac{-1}{9}}$

You can add/subtract/multiply/divide or scalar multiply Limits.

Example(s) 6: $\lim_{x\to c} f(x) = 5$ $\lim_{x\to c} g(x) = -3$

A.) $\lim_{x\to c}[5g(x)] = 5\lim_{x\to c} g(x) = 5[-3] = \boxed{-15}$

B.) $\lim_{x\to c}[f(x)+g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = 5 + -3 = \boxed{2}$

C.) $\lim_{x\to c}[f(x)g(x)] = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x) = 5[-3] = \boxed{-15}$

D.) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \boxed{\frac{5}{-3}}$

Practice Before We Leave Today:

1. $\lim_{x\to 0}\frac{\frac{1}{x+5}-\frac{1}{5}}{x}$

$\frac{5(1)}{5(x+5)} - \frac{1(x+5)}{5(x+5)}$

$\lim_{x\to} \frac{\frac{\cancel{5} - x \cancel{-5}}{5(x+5)}}{x}$

$\lim \frac{-\cancel{x}}{5(x+5)} \cdot \frac{1}{\cancel{x}}$ $\lim_{x\to 0}\frac{-1}{5(x+5)} = \frac{-1}{5(0+5)} = \boxed{\frac{-1}{25}}$

2. $\lim_{x\to\frac{\pi}{4}} \sin x$

$\sin \pi/4$

$= \boxed{\sqrt{2}/2}$

3. $\lim_{x\to -3}\frac{x^2-3x-18}{x+3}$

$\frac{\cancel{(x+3)}(x-6)}{\cancel{x+3}}$

$(x-6)$

$-3-6 = \boxed{-9}$

4. $\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$

$\frac{\sqrt{x+2}-\sqrt{2}}{x} \cdot \frac{(\sqrt{x+2}+\sqrt{2})}{(\sqrt{x+2})+\sqrt{2})} = \frac{\cancel{x}}{\cancel{x}(\sqrt{x+2}+\sqrt{2})}$

$\lim_{x\to 0}\frac{1}{\sqrt{x+2}+\sqrt{2}} = \frac{1}{\sqrt{0+2}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{2}}$

$= \boxed{\frac{1}{2\sqrt{2}}}$

5. $\lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h}$

$\frac{3(x+h)(x+h)-3x^2}{h}$ $\frac{3(x^2+2xh+h^2)-3x^2}{h}$

$\frac{\cancel{3x^2}+6xh+3h^2\cancel{-3x^2}}{h}$

$\frac{6xh+3h^2}{h}$ $\frac{\cancel{h}(6x+3h)}{\cancel{h}}$

$6x+3h$

$6x+3(0)$

$\boxed{6x}$