Notes: Curve Sketching

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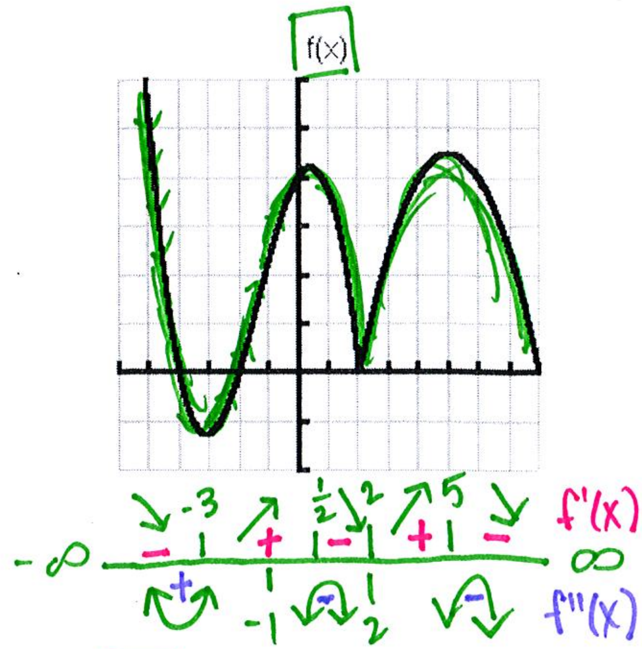
Ex: 1 Given the graph of f (x) find the following intervals or x values where
(Estimate to the nearest half unit.)



a. $f'(x)$ is positive

$(-3, \frac{1}{2}) \cup (2, 5)$

critical numbers

b. $f'(x) = 0$

$x = -3, \frac{1}{2}, 5$

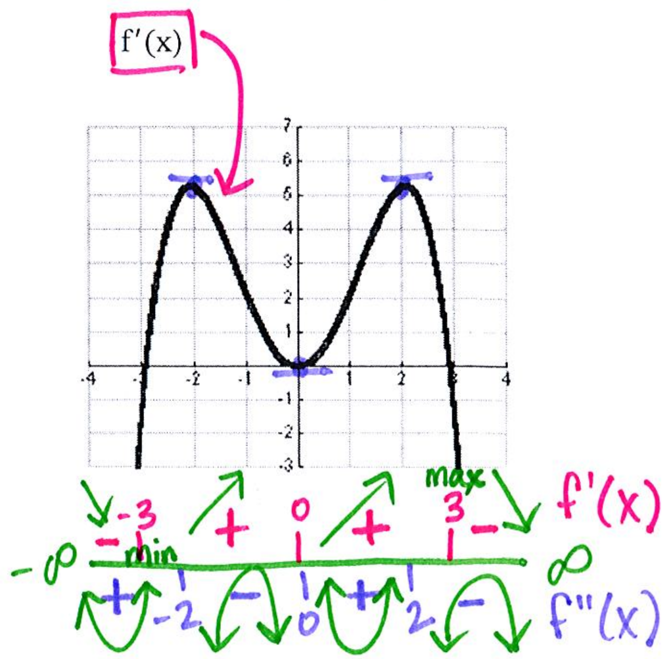
c. $f'(x)$ is undefined

$x = 2$

d. $f''(x)$ is positive

$(-\infty, -1)$

Ex: 2 Given the graph of $f'(x)$ find the following intervals or x values where
(Estimate to the nearest half unit.)



a. f(x) is increasing

$(-3, 3)$

b. f (x) has a horizontal tangent

when $f'(x) = 0$ $x = -3, 0, \& 3$

c. f (x) is concave up

$(-\infty, -2) \cup (0, 2)$

d. f (x) has a point of inflection

$x = -2, 0, \& 2$

Reminder:

VA: Set denominator of $f(x) = 0$ and solve for x.

HA: Compare the degree in the top/bottom of $f(x)$.

J-Lo=$\frac{degree\ in\ top\ smaller}{degree\ bottom}$ then y=0 Marilyn=$\frac{degree\ in\ top\ equal}{degree\ in\ bottom}$ then y=leading coefficients

Dolly=$\frac{degree\ in\ top\ larger}{degree\ in\ bottom}$ then no HA... Slant if the degree is bigger by one in numerator.

x-intercept: Set numerator of $f(x) = 0$ and solve for x.

y-intercept: Plug 0 in for x and solve for y.

Ex 3: Consider the function f, whose formula and derivatives are given by

$$\boxed{f(x)=\frac{x^2-4}{(x-1)^2}},\quad \boxed{f'(x)=\frac{-2x+8}{(x-1)^3}},\quad \boxed{f''(x)=\frac{4x-22}{(x-1)^4}}$$

a. Find and describe all of the vertical and horizontal asymptotes of this function, if any. Justify.

VA: $(x-1)^2=0$
$x-1=0$
$\boxed{x=1}$

HA: $\frac{\text{deg top}=2}{\text{deg bottom}=2}=\boxed{y=1}$

b. Find all of the roots of this function, if any.

$\boxed{\text{x int}}$ $x^2-4=0$
$x^2=4$
$x=\pm2$
$\boxed{(2,0)\quad(-2,0)}$

$\boxed{\text{y int}}$ $y=\frac{0^2-4}{(0-1)^2}=\frac{-4}{1}=-4$

$(0,-4)$

c. Find and classify all of the local extrema of this function if any. Show justification.

$-2x+8=0$
$2x=8$
$x=4$

$(x-1)^3=0$
$x-1=0$
$\cancel{x=1}$ V.A.

$f'(0)=-$
$f'(3)=+$
$f'(5)=-$

$x=4$ is a max because $f'(x)$ changes from pos to neg. at $x=4$.

d. Find all of the inflection points of this function, if any. Show justification.

$4x-22=0$
$4x=22$
$x=5.5$

$(x-1)^4=0$
$x-1=0$
$\cancel{x=1}$ V.A.

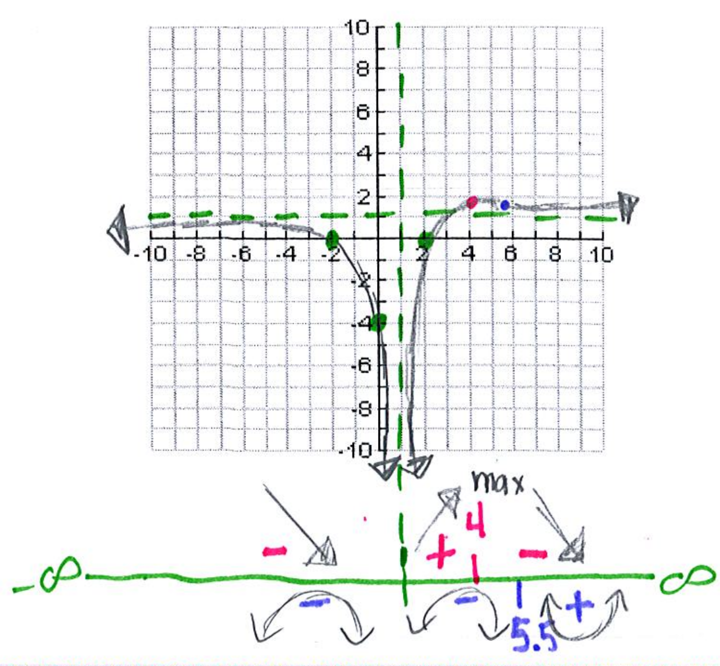
$f''(0)=-$
$f''(4)=-$
$f''(6)=+$

$x=5$ is POI b.c. $f''(x)$ changes sign there.

e. Sketch the function and include all the features above.

max$(4, 1.\bar{3})$

POI$(5.5, 1.296)$



Ex 4: Consider the function f, whose formula and derivatives are given by

$$\boxed{f(x)=\frac{x-3}{x^2-1}},\ \boxed{f'(x)=\frac{-(x^2-6x+1)}{(x^2-1)^2}},\ \boxed{f''(x)=\frac{2(x^3-9x^2+3x-3)}{(x^2-1)^3}}$$

a. Find and describe all of the vertical and horizontal asymptotes of this function, if any. Justify.

$f(x)=\frac{x-3}{x^2-1}$

VA: set bottom = 0 and solve for x

$x^2-1=0$

$x^2=1$ $\boxed{x=\pm 1}$

HA: compare degree

$\frac{\text{deg top}=1}{\text{deg bottom}=2}$ $\boxed{y=0}$

a. Find all of the roots of this function, if any.

$f(x)=\frac{x-3}{x^2-1}$

xint: set top = 0 and solve

$x-3=0$ $\boxed{(3,0)}$

$x=3$

yint: plug 0 in for x and solve for y

$y=\frac{0-3}{0^2-1}=\frac{-3}{-1}=3$ $\boxed{(0,3)}$

b. Find and classify all of the local extrema of this function if any. Show justification.

$f'(x)=\frac{-(x^2-6x+1)}{(x^2-1)^2}$

set top & bottom = 0 and solve

$0=-(x^2-6x+1)$

$\boxed{x=.172}$ & $\boxed{x=5.828}$

$\boxed{\text{min}}$ b.c. that is where f'(x) changes from neg. to pos.

max: because where f'(x) changes from positive to negative

$0=(x^2-1)^2$

$x^2-1=0$

$x^2=1$

$\cancel{x=\pm 1}$ V.A. are not extrema

c. Find all of the inflection points of this function, if any. Show justification.

$f''(x)=\frac{2(x^3-9x^2+3x-3)}{(x^2-1)^3}$

Set top & bottom = 0 and solve for x

$0=2(x^3-9x^2+3x-3)$

$\boxed{x=8.695}$

Because $f''(8.695)=0$ and f''(x) changes sign there.

$(x^2-1)^3=0$ $\cancel{x=\pm 1}$

$x^2-1=0$ V.A. are not P.O.I

d. Sketch the function and include all the features above.

min (.172, 2.914)

max (5.828, .086)

POI (8.695, .076)

$f(-2)=-1.\overline{6}$