Day 1 Derivatives(1)

Notes: Power Rule and Graphs of Functions and Their Derivatives

$\frac{d}{dx}$ ⟶ Means take a derivative with respect to x.

$f'(x)$ ⟶ Means take a derivative of $f(x)$.

$\frac{d}{dt}$ ⟶ Means take a derivative with respect to t.

y' ⟶ Means take a derivative.

$8^{5/3} = (\sqrt[3]{8})^5 = (2)^5 = 32$

Algebra you should know:

$$m^{\frac{a}{b}} = \sqrt[b]{m^a} = \left(\sqrt[b]{m}\right)^a$$

$$m^{-a} = \frac{1}{m^a} \quad \text{and} \quad \frac{1}{m^a} = m^{-a}$$

Rules of Exponents PC22	
$m^{\frac{a}{b}} =$	$m^{\frac{a}{b}} = \sqrt[b]{m^a} = (\sqrt[b]{m})^a$
$m^{-a} =$	$m^{-a} = \frac{1}{m^a}$

Power Rule: For all exponents n

- $\frac{d}{dx}[x^n] = \underline{n \cdot x^{n-1}}$

Multiply by the exponent and drop the degree by one.

- $\frac{d}{dx}[\text{Constant}] = \underline{0}$

Power Rule D5	
$\frac{d}{dx}[x^n] =$	$\frac{d}{dx}[x^n] = n \cdot x^{n-1}$

D6	
$\frac{d}{dx}[Constant] =$	$\frac{d}{dx}[\text{Any }\#] = 0$

D7	
$\frac{d}{dx}[e^x] =$	$\frac{d}{dx}[e^x] = e^x$
$\frac{d}{dx}[e^{AT}] =$	$\frac{d}{dx}[e^{AT}] = e^{AT} \cdot \frac{d}{dx}[AT]$
$AT = anything$	

Example(s) One:

A. $3x^2 = \underline{6x}$

B. $4x^3 = \underline{12x^2}$

C. $5x^1 = \underline{5x^0 = 5}$

D. $2x^4 = \underline{8x^3}$

E. $x^{\frac{5}{2}}$ $-\frac{2}{2}$ $= \underline{\frac{5}{2}x^{3/2}}$

F. $3x^{\frac{1}{2}} = \underline{\frac{3}{2}x^{-1/2}}$

G. $2x^4\Big|_{x=-2} = \underline{8x^3} = 8(-2)^3 = 8(-8)$

$= \boxed{-64}$

Day 1 (Cont.)

H. $x^{\frac{5}{2}}\Big|_{x=4} = \underline{\frac{5}{2}x^{3/2}} = \frac{5}{2}(\sqrt{4})^3 = \frac{5}{\cancel{2}}(\cancel{8})^{4} = \boxed{20}$

D8

What is the sum and constant multiple rule of differentiable Functions.

<u>Linearity Rules</u>: Assume that f and g are differentiable functions.

- Sum Rule: $(f+g)' =$
- Constant Multiple Rule: $(cf)' =$

Example(s) Two: A. $3x^2 + 4x - 8 = \underline{6x + 4}$

0

B. $5x^3 - 4x^2 + 3x + 5 - e^x = \underline{15x^2 - 8x + 3 - e^x}$

Example Three: $f(m) = \sqrt[4]{m} + \sqrt[5]{m} + \sqrt[6]{m^7}$ find: $f'(m)$ Rewrite w/ exponents & then take derivative

$f(m) = m^{1/4} + m^{1/5} + m^{7/6}$

$f'(m) = \frac{1}{4}m^{-3/4} + \frac{1}{5}m^{-4/5} + \frac{7}{6}m^{1/6}$

Example Four: $f(x) = \frac{1}{x^{\frac{2}{3}}}$ find $f'(x)$ Rewrite w/out denominator.

$f(x) = x^{-2/3}$

$f'(x) = -\frac{2}{3}x^{-5/3}$

Example Five: $f(x) = (3x+4)^2$ find $f'(x)$ Multiply then take derivative

$f(x) = 9x^2 + 24x + 16$

$f'(x) = 18x + 24$

Example Six: $f(x) = \sqrt{x}(x^2 + 2x + 3)$ find $f'(x)$ Multiply $x^{1/2}$ to each term (multiply "like" bases you add exponents)

$f(x) = x^{1/2}(x^{\frac{4}{2}} + 2x^{\frac{2}{2}} + 3)$

$f(x) = x^{5/2} + 2x^{3/2} + 3x^{1/2}$

$f'(x) = \frac{5}{2}x^{3/2} + \cancel{2}\left(\frac{3}{\cancel{2}}\right)x^{1/2} + 3\left(\frac{1}{2}\right)x^{-1/2} = \frac{5}{2}x^{3/2} + 3x^{1/2} + \frac{3}{2}x^{-1/2}$

Day 1 (Cont.)

Example Seven: $g(t)=\dfrac{t^4+6t^3-9t^2+5t}{t}$ Find $g'(t)$

Get rid of division (divide "like" bases you subtract exponents)

$$g(t)=\frac{t^4}{t}+\frac{6t^3}{t}-\frac{9t^2}{t}+\frac{5t}{t}$$

$$g(t)=t^3+6t^2-9t+5 \qquad g'(t)=3t^2+12t-9$$

Example Eight: $y=\dfrac{x^4+3x^3-2x^2+5}{x^{\frac{1}{2}}}$ Find y'

$$y=\frac{x^{4\frac{8}{2}}}{x^{1/2}}+\frac{3x^{3\frac{6}{2}}}{x^{1/2}}-\frac{2x^{2\frac{4}{2}}}{x^{1/2}}+\frac{5}{x^{1/2}}$$

$$y=x^{7/2}+3x^{5/2}-2x^{3/2}+5x^{-1/2}$$

$$y'=\frac{7}{2}x^{5/2}+3\left(\frac{5}{2}\right)x^{3/2}-\cancel{2}\left(\frac{3}{\cancel{2}}\right)x^{1/2}+5\left(\frac{-1}{2}\right)x^{-3/2}$$

$$y'=\frac{7}{2}x^{5/2}+\frac{15}{2}x^{3/2}-3x^{1/2}-\frac{5}{2}x^{-3/2}$$

Example Nine: Given $f(x)=e^x+3$ find the equation of the tangent line at x=0.

Point: $(0, 4)$ $\quad f(x)=e^x+3 \quad f(0)=e^0+3=1+3=4$

Slope: $f'(0)=$ 1

$$f(x)=e^x+3$$
$$f'(x)=e^x$$
$$f'(0)=e^0=1$$

$$y-y_1=m(x-x_1)$$
$$\boxed{y-4=1(x-0)}$$

Example Ten: Given $f(x)=3x^2+4x-8$ find the equation of the tangent line at x=2.

Point: $(2, 12)$ $\quad f(2)=3(2)^2+\cancel{4(2)-8}=12$

Slope: $f'(2)=$ 16

$$f(x)=3x^2+4x-8$$
$$f'(x)=6x+4$$
$$f'(2)=6(2)+4$$
$$f'(2)=16$$

Point/Slope Form

$$\boxed{y-12=16(x-2)}$$

$$y-12=16(x-2)$$
$$y-\cancel{12}=16x-32$$
$$+\cancel{12} \qquad +12$$

Slope/Intercept Form

$$\boxed{y=16x-20}$$ (y-intercept)

Standard Form

$$\boxed{\begin{array}{l}16x-y-20=0\\16x-y=20\end{array}}$$

- x & y are on same side of equation
- x is always positive
- No Fractions