Notes: Definite Integrals

- Sometimes you find an approximate (past 2 days)
- Sometimes you can find the exact.

→ When you integrate you are finding the **Area** between the function and the $x-axis$.

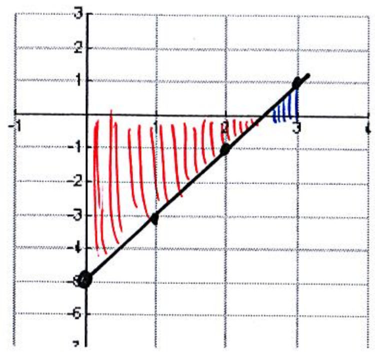
→ Area **Above** the $x-axis$ is **positive**.

→ Area **Below** the $x-axis$ is **negative**.

Example One: $\int_0^3 2x-5\ dx = \boxed{-6}$

$-\frac{1}{2}\left(\frac{5}{2}\right)(5)+\frac{1}{2}\left(\frac{1}{2}\right)(1)$

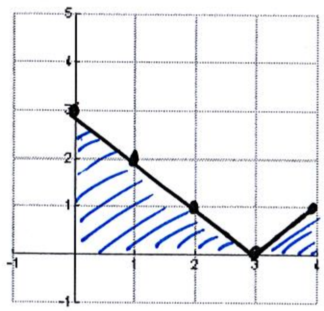
$-\frac{25}{4}+\frac{1}{4}$



Example Two: $\int_0^4 |3-x|\ dx = \boxed{5}$

$\frac{1}{2}(3)(3)+\frac{1}{2}(1)(1)$

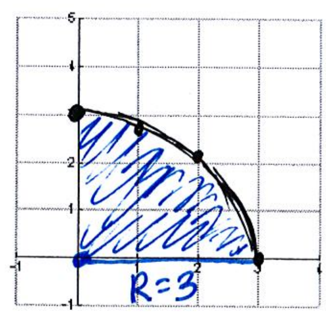
$\frac{9}{2}+\frac{1}{2}$



Example Three: $\int_0^3 \sqrt{9-x^2}\ dx = \boxed{\frac{9\pi}{4}}$

$\frac{1}{4}\pi R^2$

$\frac{1}{4}\pi 9$



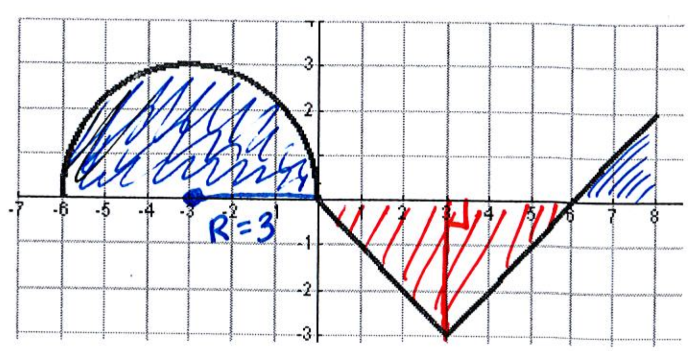
Example Four:

A. $\int_{-6}^{0} f(x)\,dx \quad \frac{1}{2}\pi R^2 = \boxed{\frac{9\pi}{2}}$

B. $\int_0^6 f(x)\,dx \quad -\frac{1}{2}(6)(3) = \boxed{-9}$

C. $\int_6^8 f(x)\,dx \quad \frac{1}{2}(2)(2) = \boxed{2}$

D. $\int_6^8 f(x)\,dx = \frac{9\pi}{2} - 9 + 2 = \boxed{\frac{9\pi}{2}-7}$



Rules for Integration

1. $-\int_a^b f(x)\,dx=$ $\underline{\int_b^a f(x)dx}$

2. $\int_a^a f(x)\,dx=$ $\underline{0}$

I-1	I-2
$-\int_a^b f(x)\,dx=$	$\int_a^a f(x)\,dx$

Example 5: $\int_0^1 f(x)dx = 2$ $\quad \int_0^2 f(x)dx = 8$ $\quad \int_1^4 f(x)dx = 4$

A. $\int_0^2 f(x)\,dx = \boxed{8}$

B. $\int_2^2 f(x)\,dx = \boxed{0}$

C. $\int_1^2 f(x)\,dx =$ $\int_0^2 f(x)dx - \int_0^1 f(x)dx = 8-2 = \boxed{6}$

D. $\int_0^4 f(x)\,dx =$ $\int_0^1 f(x)dx + \int_1^4 f(x)dx = 2+4 = \boxed{6}$

E. $\int_2^4 f(x)\,dx =$ $\int_0^4 f(x)dx - \int_0^2 f(x)dx = 6-8 = \boxed{-2}$

Example 6: $\int_0^5 f(x)dx = 8$ $\quad \int_0^5 g(x)dx = -2$

A. $\int_0^5 f(x) + g(x)\,dx =$ $8+(-2) = \boxed{6}$

B. $\int_0^5 3f(x) - 2g(x)\,dx =$ $3(8)-2(-2) = \boxed{28}$

C. $\int_0^5 5f(x)\,dx =$ $5(8) = \boxed{40}$

D. $\int_5^0 g(x)\,dx =$ $-\int_0^5 g(x)dx = --2 = \boxed{2}$

Now you try ☺ $\int_{-2}^0 f(x)dx = 6$ $\quad \int_2^6 f(x)dx = 8$ $\quad \int_6^{10} f(x)dx = -7$

A. $\int_0^2 f(x)\,dx =$

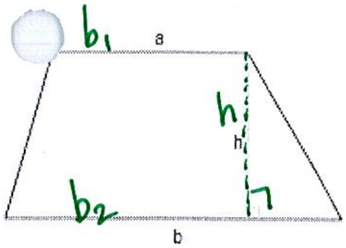
B. $\int_{10}^6 f(x)\,dx =$

C. $\int_{-2}^{10} f(x)\,dx =$

D. $\int_{-2}^{-2} f(x)\,dx =$

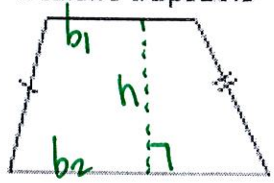
Notes: Approximating Using Trapezoidal Rule

Trapezoid: *A quadrilateral which has at least one pair of parallel sides.*

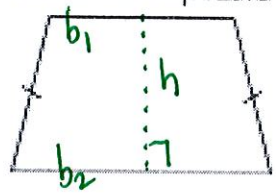


a // b: bases
h: altitude

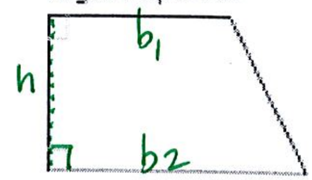
Scalene trapezoid



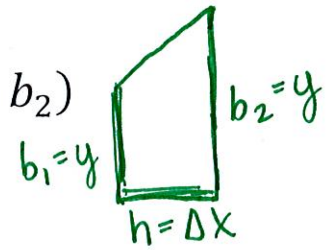
Isosceles trapezoid



Right trapezoid

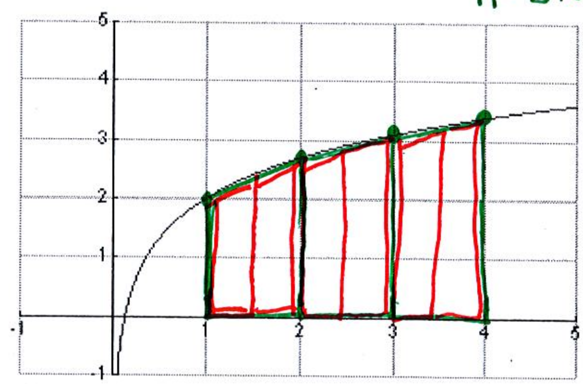


Trapezoids we will use will be the right trapezoid. The $Area = \frac{h}{2}(b_1 + b_2)$



Example One: Given $f(x) = \ln(x) + 2, \quad [1,4]$

A. Approximate the Area with T_3



$$\text{height} = \frac{4-1}{3} = \frac{3}{3} = 1$$

$$A = \frac{h}{2}[b_1 + b_2]$$

$$\frac{1}{2}[f(1) + f(2) + f(2) + f(3) + f(3) + f(4)]$$

$$\frac{1}{2}[2 + 2(2.6931) + 2(3.0986) + 3.3863]$$

$$= 8.484 \text{ OR } 8.485$$

B. Approximate the Area with T_6

$$\text{height} = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$A = \frac{h}{2}[b_1 + b_2]$$

$$\frac{.5}{2}[\underline{f(1) + f(1.5)} + \underline{f(1.5) + f(2)} + \underline{f(2) + f(2.5)} + \underline{f(2.5) + f(3)} + \underline{f(3) + f(3.5)} + \underline{f(3.5) + f(4)}]$$

$$.25[2 + 2(2.4055) + 2(2.6931) + 2(2.9163) + 2(3.0986) + 2(3.2528) + 3.3863]$$

$$= 8.529 \text{ OR } 8.53$$