day 3

AP Calculus-AB
Notes: One sided Limits

Limits: What is the y-value at some x-location (Graph Behavior)

How you write a limit:
$\lim_{x \to c} f(x) = L$

How you say a limit:
The limit as x-approaches c of f(x) is equal to L.

Right Handed Limit:
$\lim_{x \to c^+} f(x) = L$

Left Handed Limit:
$\lim_{x \to c^-} f(x) = L$

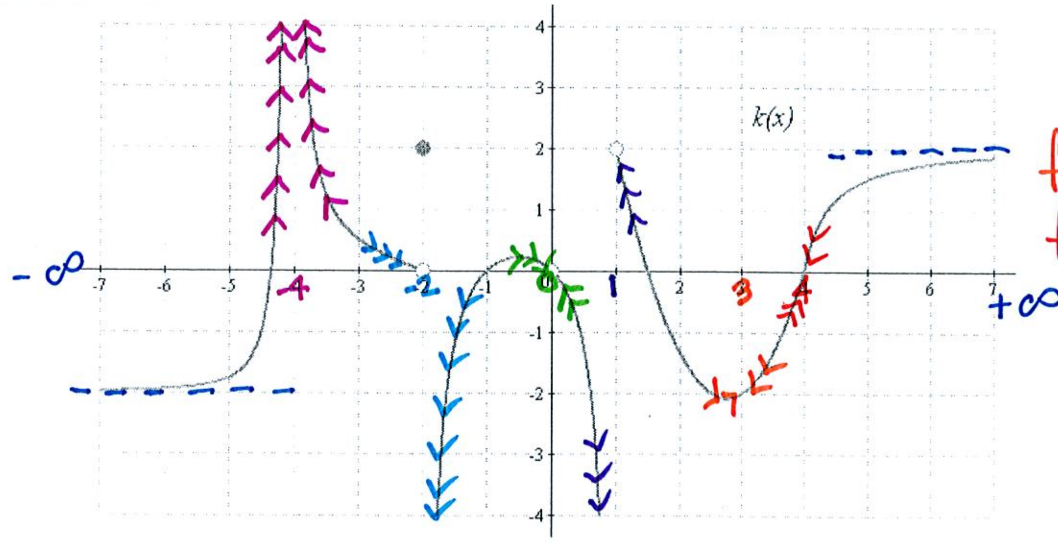
For a limit to exist:
$\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$ [The Right handed limit and the left handed limit must equal each other]

* If not equal the $\lim_{x \to c} f(x) =$ dne (does not exist)

3 Ways to Evaluate Limits:
1. Graphically
2. Numerically (Make table) $x \mid y$
3. Algebraically

Graphically

Example 1: Use the graph of $k(x)$ below to evaluate the following expressions.



$f(3) = -2$
$f(4) = 0$

$\lim_{x \to -4^-} k(x) = +\infty$	$\lim_{x \to -4^+} k(x) = +\infty$	$\lim_{x \to -4} k(x) = +\infty$	$k(-4) =$ undefined
$\lim_{x \to -2^-} k(x) = 0$	$\lim_{x \to -2^+} k(x) = -\infty$	$\lim_{x \to -2} k(x) =$ dne	$k(-2) = 2$
$\lim_{x \to 0^-} k(x) = 0$	$\lim_{x \to 0^+} k(x) = 0$	$\lim_{x \to 0} k(x) = 0$	$k(0) =$ undefined
$\lim_{x \to 1^-} k(x) = -\infty$	$\lim_{x \to 1^+} k(x) = 2$	$\lim_{x \to 1} k(x) =$ dne	$k(1) =$ undefined
$\lim_{x \to 3^-} k(x) = -2$	$\lim_{x \to 3^+} k(x) = -2$	$\lim_{x \to 3} k(x) = -2$	$\lim_{x \to \infty} k(x) = 2$
$\lim_{x \to 4^-} k(x) = 0$	$\lim_{x \to 4^+} k(x) = 0$	$\lim_{x \to 4} k(x) = 0$	$\lim_{x \to -\infty} k(x) = -2$

Example 2:

Given the graph of f to the right, find the following:

$\lim_{x\to 0^-} f(x) = -\infty$

$\lim_{x\to 0^+} f(x) = +\infty$

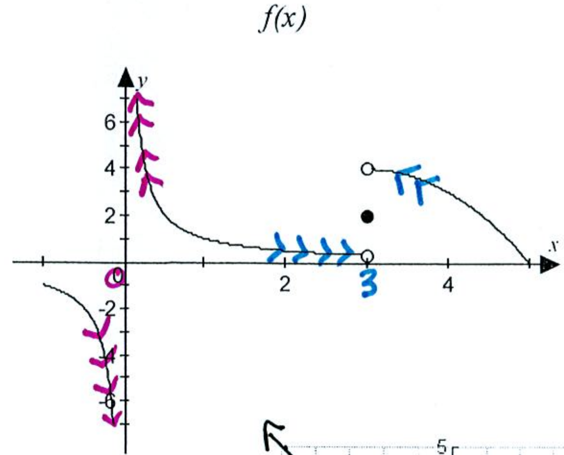
$\lim_{x\to 0} f(x) = \text{dne}$

$\lim_{x\to 3^-} f(x) = 0$

$\lim_{x\to 3^+} f(x) = 4$

$\lim_{x\to 3} f(x) = \text{dne}$

$f(3) = 2$



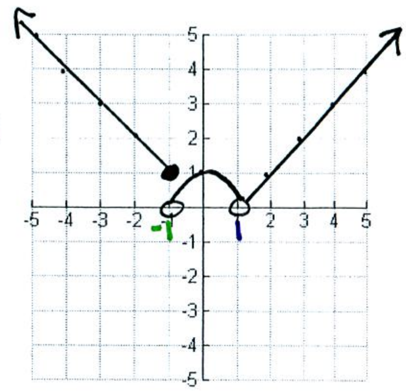
Example 3:

Let $g(x) = \begin{cases} -x & \text{if } x \le -1 \\ 1-x^2 & \text{if } -1 < x < 1 \\ x-1 & \text{if } x > 1 \end{cases}$.

$x \le -1$	$-x$
-1	1
-2	2
-3	3
-4	4

$-1<x<1$	$1-x^2$
-1	0
0	1
1	0

$x>1$	$x-1$
1	0
2	1
3	2
4	3



Find $\lim_{x\to -1^-} g(x)$, $\lim_{x\to -1^+} g(x)$, $\lim_{x\to -1} g(x)$, $\lim_{x\to 1^-} g(x)$, $\lim_{x\to 1^+} g(x)$, and $\lim_{x\to 1} g(x)$.

$= 1$ $\quad = 0$ $\quad = \text{dne}$ $\quad = 0$ $\quad = 0$ $\quad = 0$

$f(-1) = 1$ $\qquad f(1) = \text{undefined}$

Numerically

Example 4: Find the one-sided and two sided limits as x approaches -1 for the functions $h(x)$, $p(x)$, and $r(x)$ given the following table of values.

from left → -1 ← from Right

y-values / x-

x	−1.1	−1.003	−1.0001	−0.9999	−0.8762	−0.6522
$h(x)$	89	677	5009	22.003	2.088	2.113
$p(x)$	16.222	16.111	16.002	15.999	15.802	15.777
$r(x)$	−99	−999	−9999	−8853	−871	−86

16 16

$-\infty$ $-\infty$

$\lim_{x\to -1} h(x) = \text{dne}$

$\lim_{x\to -1} p(x) = 16$

$\lim_{x\to -1} r(x) = -\infty$

Example 5: $\lim_{x\to -1} \frac{x^2-2x-3}{x+1} = -4$

from left → ← from Right

-1.5	-1.1	-1.01	-1.001	-1	-.999	-.99	-.9	-.5
-4.5	-4.1	-4.01	-4.001		-3.999	-3.99	-3.9	-3.5

-4 -4

Example 6: $\lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x} = .353$

-.5	-.1	-.01	-.001	0	.001	.01	.1	.5
.3789	.378	.354	.353		.3535	.353	.349	

Example 7: $\lim_{x\to 2} x^2 - x + 7 = 9$

1.5	1.9	1.99	1.999	2	2.001	2.01	2.1	2.5
7.75	8.71	8.9701	8.997	9	9.003	9.0301	9.31	10.75