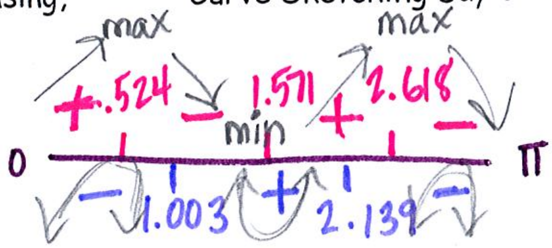
Notes: Critical Numbers, Intervals of Increasing/Decreasing, Intervals of Concavity, & POI (2)

Example One: $f(x) = cos^2x + sinx$ on the interval $[0, \pi]$



$f(x) = [\cos x]^2 + \sin x$

Critical Numbers: $x = .524, 1.571, 2.618$

Intervals of

Increasing: $(0, .524)\ (1.571, 2.618)$

Decreasing: $(.524, 1.571)\ (2.618, \pi)$

Maximum Value: $x = .524$ $\boxed{1.25}$; $x = 2.618$ $\boxed{1.25}$

Minimum Value: $x = 1.571$ $\boxed{1}$

Possible Points of Inflection:

Intervals of

Concave Upward: $(1.003, 2.139)$

Concave Downward: $(0, 1.003) \cup (2.139, \pi)$

Point(s) of Inflection:

$(1.003, 1.132)\ (2.139, 1.132)$

$f'(x) = 2[\cos x]^1(-\sin x) + \cos x$

$0 = -2\cos x \sin x + \cos x$

$y_1 = -2\cos(x) * \sin(x) + \cos x$

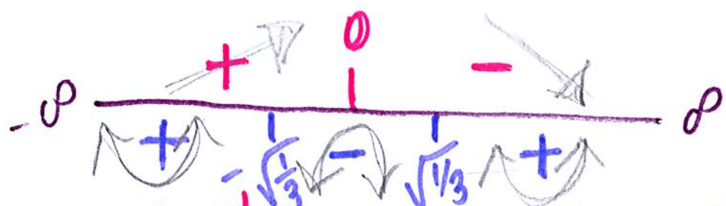
$f'(x) = -2\cos x \cdot \sin x + \cos x$

$f''(x) = -2\cos x(\cos x) + \sin x(+2\sin x) + -\sin x$

$f''(x) = -2\cos^2 x + 2\sin^2 x - \sin x$

$y_1 = -2(\cos(x))^\wedge 2 + 2(\sin(x))^\wedge 2 - \sin x$

Example Two: $y = \frac{1}{x^2+1}$



Critical Numbers: $x = 0$

Intervals of

Increasing: $(-\infty, 0)$

Decreasing: $(0, \infty)$

Maximum Value: $x = 0$ $\boxed{1}$

Minimum Value: none

Possible Points of Inflection:

Intervals of

Concave Upward: $(-\infty, -\sqrt{1/3})$ $(\sqrt{1/3}, \infty)$

Concave Downward: $(-\sqrt{1/3}, \sqrt{1/3})$

Point(s) of Inflection:

$\left(\pm\sqrt{\frac{1}{3}}, .75\right)$

$y = 1(x^2+1)^{-1}$

$y' = -1(x^2+1)^{-2}(2x)$

$y' = \frac{-2x}{(x^2+1)^2}$ $\quad y'(1) = \frac{-}{+}$ $\quad y'(-1) = \frac{+}{+}$

$-2x = 0$; $\frac{-2x}{-2} = \frac{0}{-2}$; $x = 0$

$\sqrt{(x^2+1)^2} = \sqrt{0}$; $x^2 + 1 = 0$; $\cancel{\sqrt{x^2} = \sqrt{-1}}$

garbage

$y'' = \frac{(x^2+1)^2(-2) - (-2x)2(x^2+1)^1(2x)}{(x^2+1)^4}$

$y'' = \frac{-2(x^2+1)^2 + 8x^2(x^2+1)}{(x^2+1)^4}$

$y'' = \frac{\cancel{(x^2+1)}[-2(x^2+1) + 8x^2]}{(x^2+1)^{\cancel{4}3}}$

$y'' = \frac{-2x^2 - 2 + 8x^2}{(x^2+1)^3} = \frac{6x^2 - 2}{(x^2+1)^3}$

$6x^2 - 2 = 0$

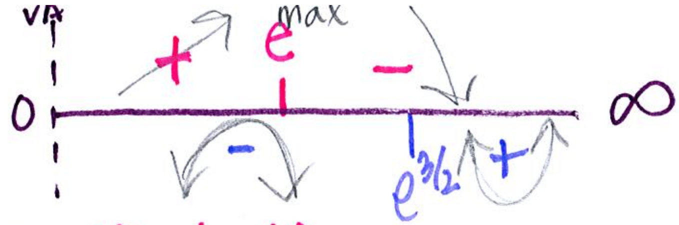
$\sqrt{x^2} = \sqrt{\frac{1}{3}}$

$x = \pm\sqrt{1/3}$

$\cancel{(x^2+1)^3 = 0}$

$y = \frac{1}{\left(\pm\sqrt{\frac{1}{3}}\right)^2 + 1} = \frac{1}{\frac{1}{3} + 1} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$

Example Three: $y = \frac{\ln x}{x}$



Critical Numbers: $x = e$

$y' = \frac{\cancel{x}\left(\frac{1}{\cancel{x}}\right) - \ln x(1)}{x^2}$

Intervals of

Increasing: $(0, e)$

Decreasing: (e, ∞)

$y' = \frac{1 - \ln x}{x^2}$

$y'' = \frac{x^{\cancel{2}1}\left(-\frac{1}{\cancel{x}}\right) - (1-\ln x)(2x)}{x^4}$

Maximum Value: $x = e$ $\boxed{\frac{1}{e}}$

Minimum Value: none

Possible Points of Inflection:

$1 - \ln x = 0$ $\quad x^2 = 0$

$e^1 = e^{\ln x}$ $\quad x = 0$

$x = e$

$\frac{-x - 2x + 2x\ln x}{x^4}$

$-3 + 2\ln x = 0$

$2\ln x = 3$

$e^{\ln x} = e^{3/2}$

$x = e^{3/2}$

Intervals of

Concave Upward: $(e^{3/2}, \infty)$

Concave Downward: $(0, e^{3/2})$

$\frac{-3x + 2x\ln x}{x^4} = \frac{\cancel{x}(-3 + 2\ln x)}{x^{\cancel{4}3}}$

$\frac{-3 + 2\ln x}{x^3}$

Point(s) of Inflection:

$(e^{3/2}, .335)$

The first derivative test: Assume that $f(x)$ is differentiable and let c be a critical point.

max

If $f'(c)$ changes from positive to negative,

Then $x = c$ is a local maximum.

If $f'(c)$ changes from negative to positive,

min

Then $x = c$ is a local minimum.

AD 7

What is the 1st derivative test?

The second derivative test: Assume that $f(x)$ is differentiable and let c be a critical point. If $f''(c)$ exists and

If $f''(c) > 0$, Then $x = c$ is a local minimum

If $f''(c) < 0$, Then $x = c$ is a local maximum

If $f''(c) = 0$, Then $x = c$ is can not be determined

AD 8

What is the 2nd derivative test?

Example Four:

Use the 1st ~~and 2nd~~ derivative test to find the extrema for $f(x) = x^4 - 8x^2 + 1$.

$f'(x) = 4x^3 - 16x$

$0 = 4x(x^2 - 4)$

$4x = 0$ $\quad x^2 - 4 = 0$

$x = 0$ $\quad x^2 = 4$

$x = \pm 2$

$f'(3) = (+)(+)$

$f'(1) = (+)(-)$

$f'(-1) = (-)(-)$

$f'(-3) = (-)(+)$

$x = \pm 2$ are minimums because they are critical numbers where $f'(x)$ changes from negative to positive.

$x = 0$ is max b.c. $f'(x)$ changes from pos to neg.

Example Five:

Use the ~~1st~~ and 2nd derivative test to find the extrema for $f(x) = x^3 - 3x^2 - 9x + 1$.

$f''(x) = 6x - 6$

$f''(-1) =$ neg max

$f''(3) =$ pos min

$x = -1$ is a max b.c. it is a critical # & $f''(-1) =$ neg.

$x = 3$ is a min b.c it is a critical & $f''(3) =$ pos.

$f'(x) = 3x^2 - 6x - 9$

$\frac{0}{3} = \frac{3x^2 - 6x - 9}{3}$

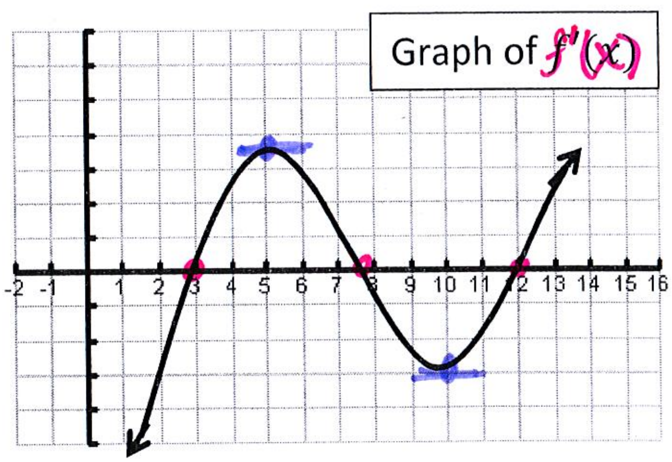
$0 = x^2 - 2x - 3$

$0 = (x+1)(x-3)$

$x = -1$

$x = 3$

Example Six:



Where does $f(x)$ have critical numbers?

$x = 3, 7.5,$ & 12

Where is $f(x)$ increasing?

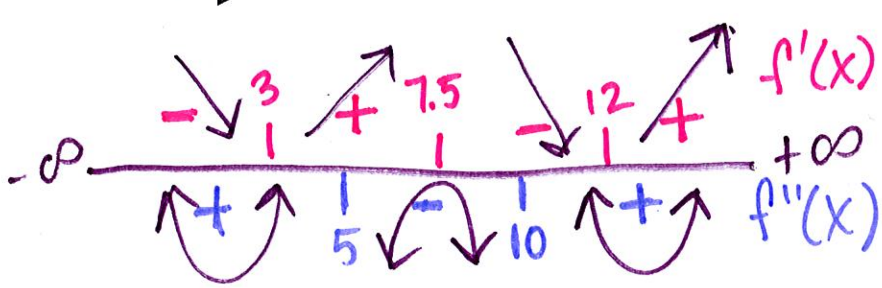
$(3, 7.5)$ $(12, \infty)$

Where is $f(x)$ decreasing?

$(-\infty, 3)$ $(7.5, 12)$

Are the critical values local minimums, maximums, or neither? min: $x = 3$ & 12

max: $x = 7.5$



Where is $f(x)$ concave upward?

$(-\infty, 5)$ $(10, \infty)$

Where is $f(x)$ concave downward?

$(5, 10)$

Where does $f(x)$ have points of inflections?

$x = 5$ & $x = 10$

What is the difference between critical numbers and points of inflection?

Critical #'s are where $f'(x) = 0$

Points of inflection are where $f''(x) = 0$ And have sign change