Notes: Critical Numbers, Intervals of Increasing/Decreasing, Intervals of Concavity, & POI

Curve Sketching Day 2

Example One: Use the graph of f to find the following:

Intervals of

Increasing: (6,10)

Decreasing: $(-\infty, 6)(10, \infty)$

Maximum: x=10 (location) Value: 5.5

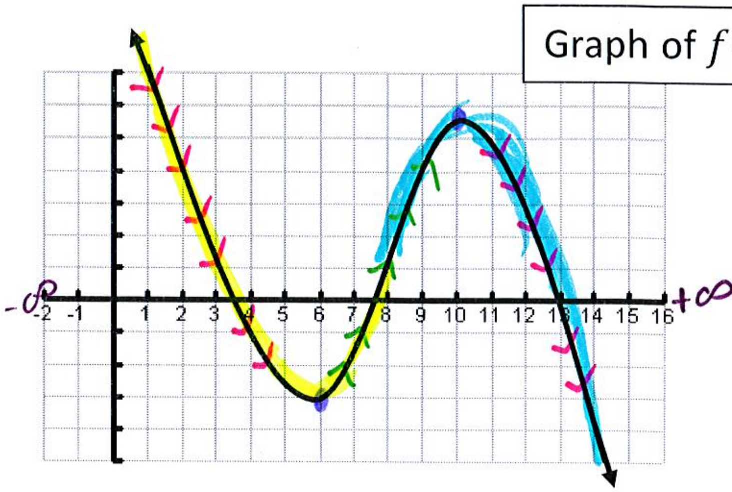
Minimum: x=6 (location) value: -3

Intervals of

Concave Upward: $(-\infty, 8)$

Concave Downward: $(8, \infty)$

Graph of $f(x)$




Critical points: A number $c$ in the domain of $f$ is called a critical point

**If:** $f'(c) = 0$ $or$ $f'(c) = does\ not\ exist$

If: $f'(x) = \text{fraction} = \frac{p(x)}{q(x)}$ $p(x)=0$ where $f'(x)=0$; $q(x)=0$ where $f'(x)=$ d.n.e.

**Then:** $x = c$ is a critical point.

AD 3

Critical Numbers

Increasing/Decreasing Behavior of Function

- If $f'(x) > 0$, then $f$ is increasing.
  $f'(x)$ is pos.
- If $f'(x) < 0$, then $f$ is decreasing.
  $f'(x)$ is neg.

AD 4

How does the sign of the first derivative relate to the original function?

Increasing/Decreasing Behavior of Function

- If $f''(x) > 0$, then $f$ is concave up.
  $f''(x)$ is pos.
- If $f''(x) < 0$, then $f$ is concave down.
  $f''(x)$ is neg.

AD 5

How does the sign of the second derivative relate to the original function?

Points of Inflection: A number $c$ in the domain of $f$ is called a point of inflection

**If:** $f''(c) = 0$ $or$ $f''(c) = does\ not\ exist$

**And:** If sign change at c

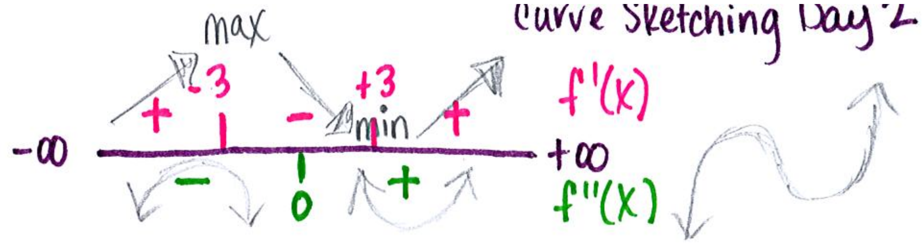
**Then:** $x = c$ is a point of inflecton

AD 6

Points of Inflection

Curve Sketching Day 2

Example Two: $f(x) = x^3 - 27x - 20$




Critical Numbers: $x = \pm 3$

Intervals of

Increasing: $(-\infty, -3) \cup (3, \infty)$

Decreasing: $(-3, 3)$

Maximum Value: $x = -3$ (location) $\boxed{34}$

Minimum Value: $x = 3$ (location) $\boxed{-74}$

Possible Points of Inflection: $x = 0$

Intervals of

Concave Upward: $(0, \infty)$

Concave Downward: $(-\infty, 0)$

Point(s) of Inflection:

$(0, -20)$

$f'(x) = 3x^2 - 27$

$0 = 3x^2 - 27$

$\frac{3x^2}{3} = \frac{27}{3}$

$x^2 = 9$

$x = \pm 3$

$f'(-5) = +$

$f'(0) = -$

$f'(265) = +$

$f''(x) = 6x$

$0 = 6x$

$x = 0$

$f''(-2) = \text{neg}$

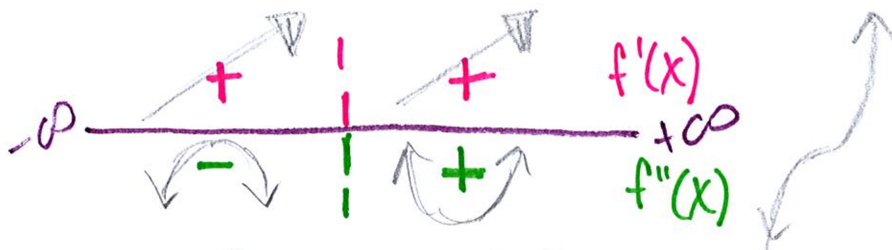
$f''(1{,}000{,}000) = \text{pos}$

$f(-3) = (-3)^3 - 27(-3) - 20 = 34$

$f(3) = (3)^3 - 27(3) - 20 = -74$

$f(0) = (0)^3 - 27(0) - 20 = -20$

Example Three: $f(x) = \frac{1}{3}x^3 - x^2 + x$




Critical Numbers: $x = 1$

Intervals of

Increasing: $(-\infty, \infty)$

Decreasing: none

Maximum Value: none

Minimum Value: none

Possible Points of Inflection: $x = 1$

Intervals of

Concave Upward: $(1, \infty)$

Concave Downward: $(-\infty, 1)$

Point(s) of Inflection:

$\left(1, \frac{1}{3}\right)$

$f'(x) = x^2 - 2x + 1$

$0 = (x-1)(x-1)$

$x - 1 = 0$

$x = 1$

$f'(2) = \text{pos}$

$f'(0) = \text{pos}$

$f''(x) = 2x - 2$

$0 = 2x - 2$

$2x = 2$

$x = 1$

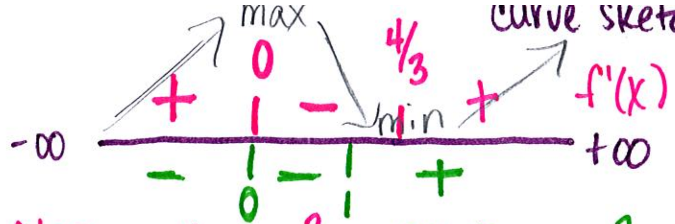
$f''(2) = \text{pos}$

$f''(0) = \text{neg}$

$f(1) = \frac{1}{3}(1)^3 - (1)^2 + 1 = \frac{1}{3}$

Curve Sketching Day 2

Example Four: $f(x) = 3x^5 - 5x^4 + 1$




Critical Numbers: $x = 0, \frac{4}{3}$

$f'(x) = 15x^4 - 20x^3$ $\quad$ $f''(x) = 60x^3 - 60x^2$

Intervals of

Increasing: $(-\infty, 0)(\frac{4}{3}, \infty)$ $\quad$ $0 = 5x^3(3x-4)$ $\quad$ $0 = 60x^2(x-1)$

Decreasing: $(0, 4/3)$ $\quad$ $5x^3 = 0$ $\quad$ $3x - 4 = 0$ $\quad$ $60x^2 = 0$ $\quad$ $x - 1 = 0$

Maximum Value: $x = 0$ $\boxed{\text{Value: } 1}$ $\quad$ $x = 0$ $\quad$ $x = \frac{4}{3}$ $\quad$ $x = 0$ $\quad$ $x = 1$

Minimum Value: $x = 4/3$ $\boxed{\text{Value: } -\frac{175}{81}}$

Possible Points of Inflection: $x = 0$ & $x = 1$

Intervals of $\quad$ $f'(-1) = (-)(-) = \text{pos}$ $\quad$ $f''(-1) = (+)(-) = \text{neg}$

Concave Upward: $(1, \infty)$ $\quad$ $f'(1) = (+)(-) = \text{neg}$ $\quad$ $f''(\frac{1}{2}) = (+)(-) = \text{neg}$

Concave Downward: $(-\infty, 1)$ $\quad$ $f'(2) = (+)(+) = \text{pos}$ $\quad$ $f''(2) = (+)(+) =$

Point(s) of Inflection:

$(1, -1)$

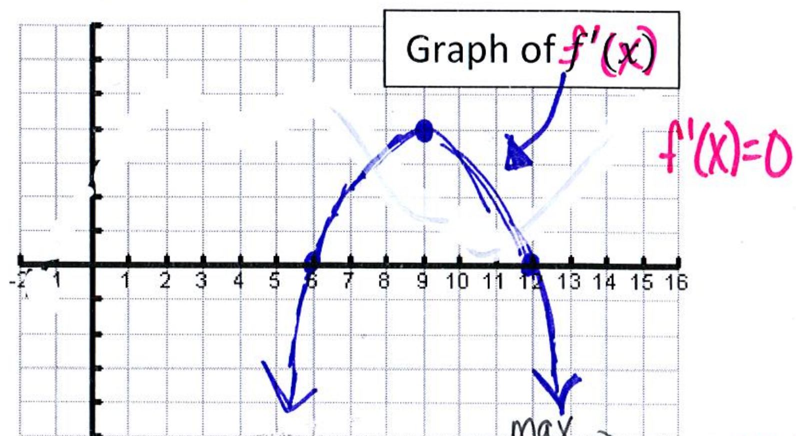
$x = 0$ not P.O.I b.c. no sign change

$f''(0) = 3(0)^5 - 5(0)^4 + 1 = 1$

$f''(4/3) = 3(\frac{4}{3})^5 - 5(\frac{4}{3})^4 + 1 = -\frac{175}{81}$

$f''(1) = 3(1)^5 - 5(1)^4 + 1 = -1$

Example Five:




Where does $f(x)$ have critical numbers? $f'(x) = 0$

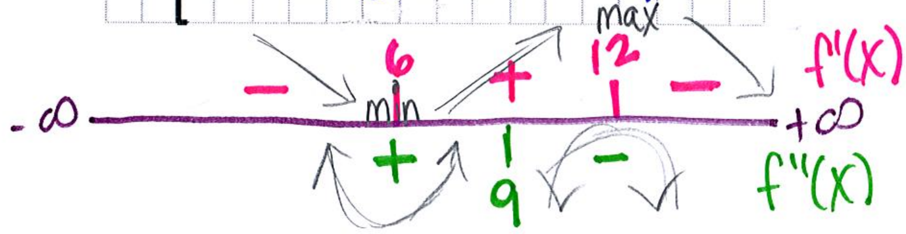
$x = 6$ & $x = 12$

Where is $f(x)$ increasing? $(6, 12)$

$f'(x) > 0$ or above x-axis

Where is $f(x)$ decreasing? $(-\infty, 6) \cup (12, \infty)$

$f'(x) < 0$ OR below x-axis




Are the critical values local minimums, maximums, or neither? $x = 6$ minimum

$x = 12$ maximum

Where is $f(x)$ concave upward?

$(-\infty, 9)$

Where is $f(x)$ concave downward?

$(9, \infty)$

Where does $f(x)$ have points of inflections? $x = 9$

What is the difference between critical numbers and points of inflection?

Find critical numbers set $f'(x) = 0$ & solve for x

Points of Inflection set $f''(x) = 0$ & solve for x And if sign change then P.O.I.