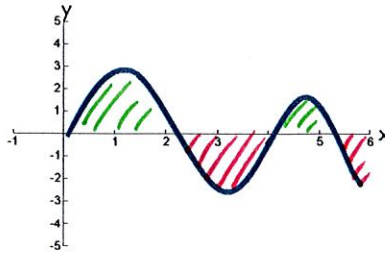
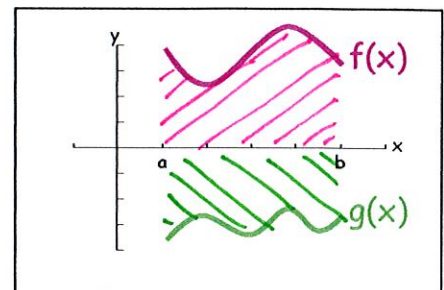
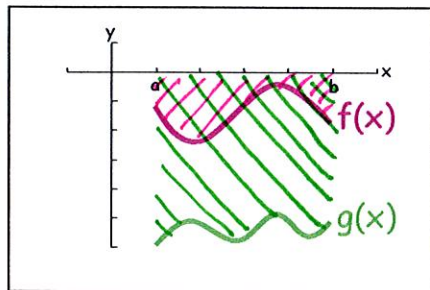
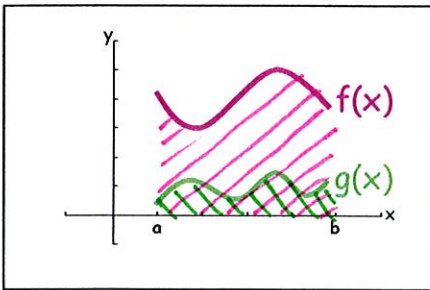


■ In the past we have found the area between the curve & the x-axis



■ Area below the x-axis is negative.
 ■ Area above the x-axis is positive.

■ Today we are find the area bounded between 2 functions.



$$\int_a^b f(x) - g(x) dx$$

■ If you get negative are, then you did something wrong.

AI-2
 How do you find the area bounded by two curves?

$$\text{Area} = \int_a^b f(x) - g(x) dx$$

$$\text{Area} = \int_a^b (\text{top function}) - (\text{bottom function}) dx$$

Example 1: Find the area of the region bounded by the graphs $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 2$.

$$\text{Area} = \int_a^b \text{top} - \text{bottom} dx$$

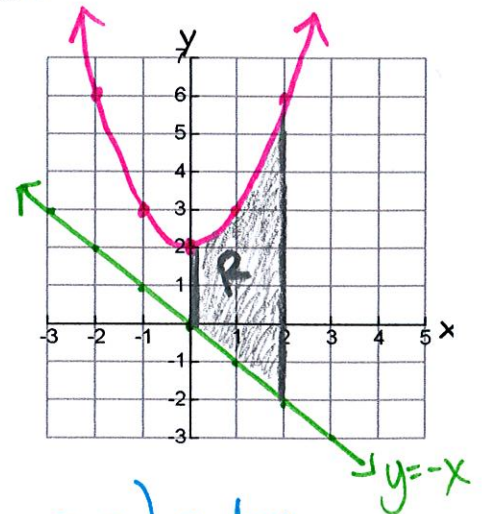
$$\int_0^2 (x^2 + 2) - (-x) dx$$

$$\int_0^2 x^2 + x + 2 dx$$

$$\left. \frac{x^3}{3} + \frac{x^2}{2} + 2x \right|_0^2$$

$$\frac{8}{3} + \frac{4}{2} + 4 - 0 - 0 - 0$$

$$\frac{8}{3} + \frac{6 \cdot 3}{3} = \boxed{\frac{26}{3}}$$



TI 89

2nd | ∫ (x^2 + x + 2, x, 0, 2) enter

TI 83/84

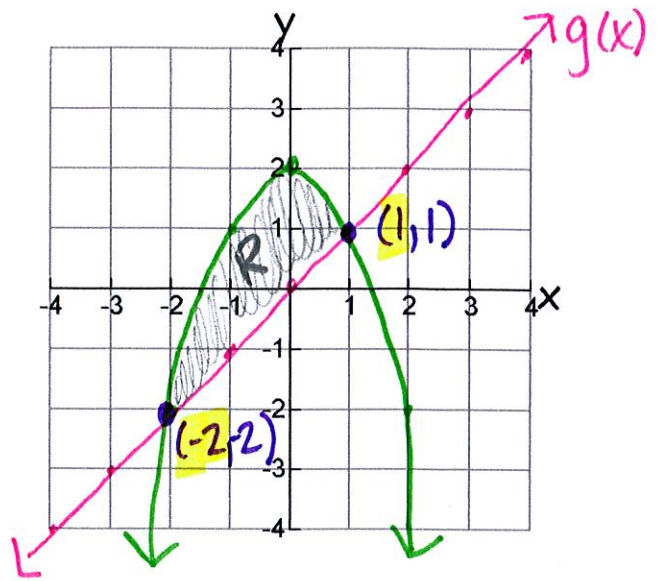
$$y_1 = x^2 + x + 2$$

2nd calc 7: ∫ (u:0 u1:2

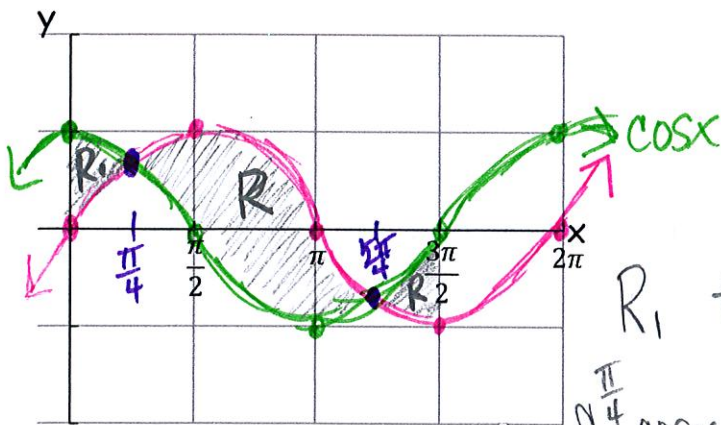
Application of Interation Dav 2

Example 2: Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$

$$\int_{-2}^1 (2 - x^2) - (x) dx = \frac{9}{2} = 4.5$$



Example 3: Find the area of the region bounded by the graphs of $f(x) = \cos x$ and $g(x) = \sin x$ on the interval $[0, \frac{3\pi}{2}]$



$$R_1 + R_2 + R_3$$

$$\int_0^{\pi/4} \cos x - \sin x dx + \int_{\pi/4}^{5\pi/4} \sin x - \cos x dx + \int_{5\pi/4}^{3\pi/2} \cos x - \sin x dx$$

$$.41421356 + 2.8284271 + .41421356$$

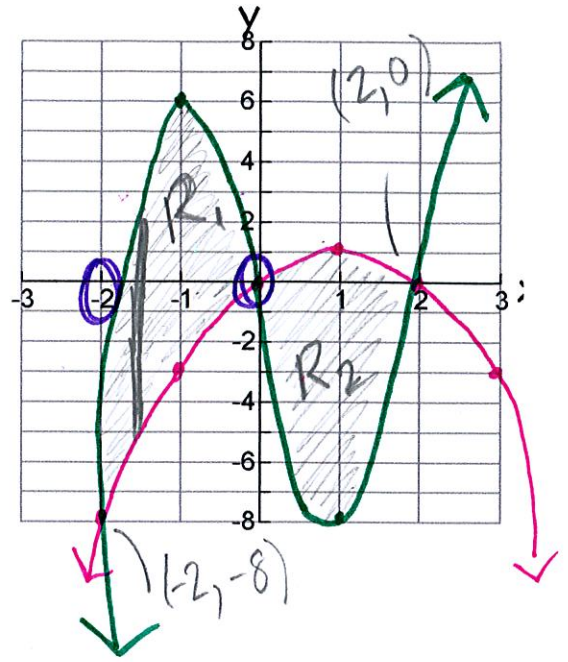
$$\boxed{3.659}$$

Application of Integration Day 2

Example 4: Find the area of the region bounded by the functions $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$.

$$\int_{-2}^0 f(x) - g(x) dx + \int_0^2 g(x) - f(x) dx$$

12 + 12
24



Sometimes you just can't do top minus bottom.

How do you find the area when top-bottom will not work?

AI-3

$$\int_{y_1}^{y_2} f(y) - g(y) dy$$

$$\int_{y_1}^{y_2} (\text{Right}) - \text{left} dy$$

Example 5: Find the area of the region bounded by

$x = 3 - y^2$ and $x = y + 1$ $y = x - 1$ $x = 3 - y^2$

$$\int_{-2}^1 (3 - y^2) - (y + 1) dy$$

84 $y_1 = 3 - x$ $2 - x - 1$

$\frac{9}{2}$ OR 4.5

$x = 3 - y^2$	y
-6	-3
-1	-2
2	-1
3	0
2	1
-1	2
-6	3

