Notes: Rolle's Theorem & Mean Value Thm. Day 1 Curve Sketching

Rolle's Theorem: Let f be a function that satisfies the following three hypothesis:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).
3. $f(a) = f(b)$ 2 x-values have the same y-value

Then there exists a number c in (a, b) such that $f'(c) = 0$.

AD 1

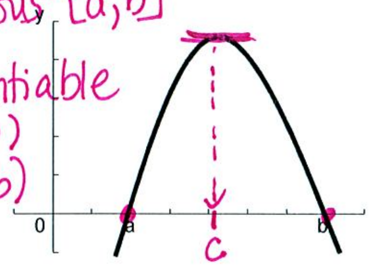
Rolle's Theorem

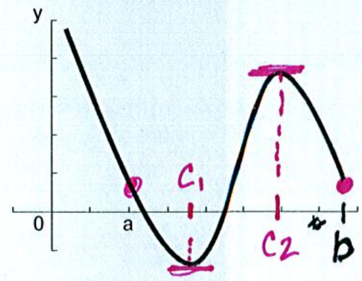
$\boxed{1.}$ continuous $[a,b]$

$\boxed{2.}$ differentiable (a,b)

$\boxed{3}$ $f(a)=f(b)$

$0=0$





Example One: Verify that the function satisfies the three hypothesis of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Thm.

A.) $f(x) = x^4 - x^2$ on $[-2,2]$ Then $f'(x)=0$ at least 1 time on $(-2,2)$

$\boxed{1.}$ Continuous $[-2,2]$

$\boxed{2.}$ Differentiable $(-2,2)$

$\boxed{3.}$ $f(-2)=f(2)$

$f(-2)=(-2)^4-(-2)^2=16-4=12$

$f(2)=(2)^4-(2)^2=16-4=12$

$4x^3-2x=0$

$2x(2x^2-1)=0$

$\frac{2x}{2}=\frac{0}{2}$ $\quad 2x^2-1=0$

$\boxed{x=0}$ $\quad 2x^2=1$

$x^2=\frac{1}{2}$

$\boxed{x=\pm\sqrt{1/2}}$

Solve In Calculator

$y_1=4x\wedge3-2x$

2nd $\boxed{\text{Trace}}$ CALC 2: zeros

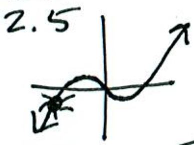
make window

x-min: -2.5

x-max: 2.5

y-min:

y-max:



$x=-.707, 0, .707$

B.) $f(x) = x^2 - 3x + 2$ on $[1,2]$

$\boxed{1.}$ Continuous $[1,2]$

$\boxed{2.}$ differentiable $(1,2)$

$\boxed{3.}$ $f(1)=f(2)=0$

$f(1)=(1)^2-3(1)+2=0$

$f(2)=(2)^2-3(2)+2=0$

Then $f'(x)=0$

$2x-3=0$

$2x=3$

$\boxed{x=\frac{3}{2}}$

Mean Value Theorem: Let f be a function that satisfies the following hypothesis:

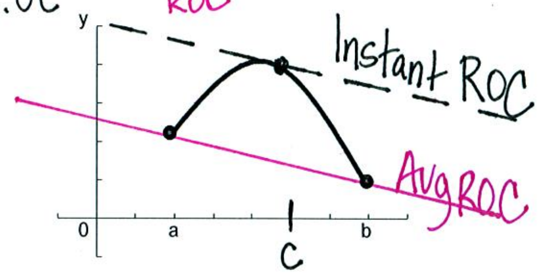
1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).

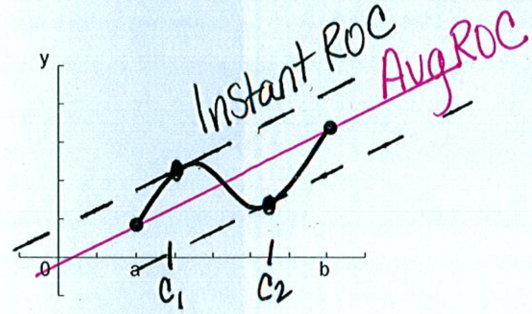
Then there exists a number c in (a, b) such that

$f'(c) = \frac{f(b)-f(a)}{b-a}$ Or $f(b) - f(a) = f'(c)(b-a)$

Instant R.OC = Average ROC

Mean Value Thm.	AD 2





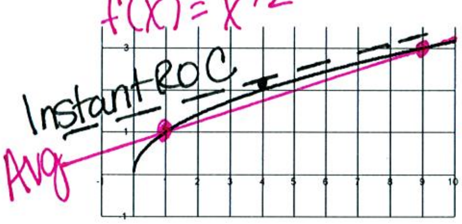
Example(s) Two: Find a point c satisfying the conclusion of the MVT for the given function and the interval:

A. $f(x) = \sqrt{x}$ $(1,9)$

$\boxed{1}$ Continuous $[1,9]$

$\boxed{2}$ differentiable $(1,9)$

$f(x) = x^{1/2}$



Then Avg ROC = Instant ROC

$\frac{f(9)-f(1)}{9-1} = \frac{1}{2}x^{-1/2}$

$\frac{3-1}{8} = \frac{1}{2\sqrt{x}}$

$\frac{2}{8} = \frac{1}{2\sqrt{x}}$

$\frac{1}{4} = \frac{1}{2\sqrt{x}}$

$\frac{2\sqrt{x}}{2} = \frac{4}{2}$

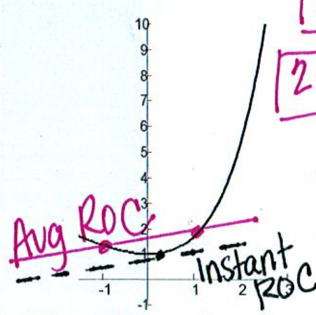
$(\sqrt{x})^2 = (2)^2$

$x = 4$

B. $f(x) = e^x - x$ $(-1,1)$

$\boxed{1.}$ Continuous $[-1,1]$

$\boxed{2.}$ Differentiable $(-1,1)$



Then Avg ROC = Instant ROC

$\frac{f(1)-f(-1)}{1-(-1)} = f'(x)$

$\frac{(e^1-1)-(e^{-1}+1)}{2} = e^x - 1$

$\frac{e^1-1-e^{-1}-1}{2} = e^x - 1$

$.1752011 = e^x - 1$

$0 = e^x - 1.1752011$

$x = .161$

Example Three: Find a point c satisfying the conclusion of the MVT for the given function and the interval:

$f(x) = x^3 - x$ on $[0,2]$

$f(2) = (2)^3 - 2 = 8 - 2 = 6$

$f(0) = (0)^3 - 0 = 0 - 0 = 0$

$\frac{f(2)-f(0)}{2-0} = 3x^2 - 1$

$\frac{6-0}{2} = 3x^2 - 1$

$3 = 3x^2 - \cancel{1}$
$+1 \quad \cancel{+1}$

$\frac{4}{3} = \frac{\cancel{3}x^2}{\cancel{3}}$

$\sqrt{x^2} = \sqrt{\frac{4}{3}}$

$x = \pm\sqrt{4/3}$

$\boxed{x = \sqrt{4/3}}$

Calculator

$3 = 3x^2 - 1$

$0 = 3x^2 - 4$

$x = 1.155$