

AI-1

Formula for Average  
Value of  $f(x)$  on  $[a, b]$

$$\text{Avg Value} = \frac{1}{b-a} \int_a^b f(x) dx$$

Example One: Find the average value of  $f(x) = \sin x$

A. On  $[0, \pi]$ 

$$\begin{aligned} & \frac{1}{\pi} \int_0^{\pi} \sin x \, dx \\ & \quad -\frac{1}{\pi} \cos x \Big|_0^{\pi} \\ & -\frac{1}{\pi} \cos \pi - -\frac{1}{\pi} \cos(0) \\ & -\frac{1}{\pi}(-1) - -\frac{1}{\pi}(1) \\ & \quad +\frac{1}{\pi} + \frac{1}{\pi} = \boxed{\frac{2}{\pi}} \end{aligned}$$

B. On  $[0, 2\pi]$ 

$$\begin{aligned} & \frac{1}{2\pi-0} \int_0^{2\pi} \sin x \, dx \\ & \quad -\frac{1}{2\pi} \cos x \Big|_0^{2\pi} \\ & -\frac{1}{2\pi} \cos(2\pi) - -\frac{1}{2\pi} \cos(0) \\ & -\frac{1}{2\pi}(1) + \frac{1}{2\pi}(1) = \boxed{0} \end{aligned}$$

Example Two: Find the average value of  $f(x) = 2x + x^2$  on  $[8, 10]$

$$\begin{aligned} & \frac{1}{10-8} \int_8^{10} 2x + x^2 \, dx \rightarrow \frac{1}{2} x^2 + \frac{1}{6} x^3 \Big|_8^{10} \\ & \frac{1}{2} \left[ \frac{2x^2}{2} + \frac{x^3}{3} \right]_8^{10} \rightarrow \frac{100}{2} + \frac{1000}{6} - \frac{64}{2} - \frac{512}{3} = \frac{36}{2} + \frac{488}{6} \\ & 18 + \frac{244}{3} = \frac{54}{3} + \frac{244}{3} = \end{aligned}$$

$$\boxed{\frac{298}{3}} \quad \therefore$$

Example Three:

A.  $\int_a^b \frac{\text{miles}}{\text{hour}} =$

$\left( \frac{1}{\text{hr} - \text{hr}} \int_a^b \frac{\text{miles}}{\text{hour}} \right) = \frac{1}{\text{hr}} \cdot \text{miles} = \frac{\text{miles}}{\text{hour}}$

B.  $\frac{1}{b-a} \int_a^b \frac{\text{miles}}{\text{hour}} =$

C.  $\int_a^b \frac{\text{gallons}}{\text{minute}} =$  gallons

D.  $\frac{1}{b-a} \int_a^b \frac{\text{gallons}}{\text{minute}} =$  gallons/minute

I ♥ Math  
-♥- Hunter