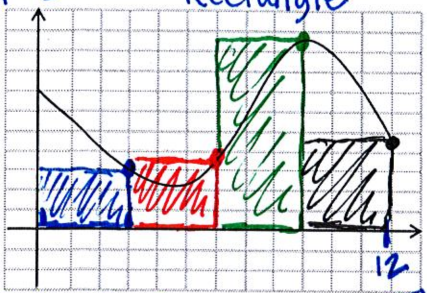
Notes: Approximating The Area Under a Curve: Integration: Day 1

We can approximate area by making rectangles. We can use right endpoints, left endpoints, or midpoints of these rectangles.

Example 1: Estimate the area under the curve . (Some answers may differ if you are given a picture and asked for an estimate.) Right endpoints

A. R_4 [0,12] ← # Rectangle

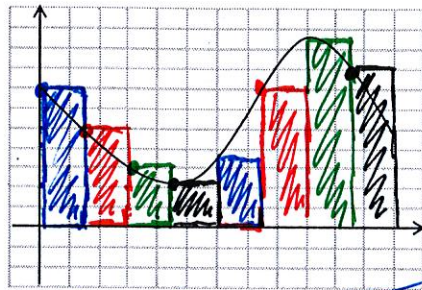


width = $\frac{12-0}{4} = 3$ $\boxed{A \approx 61.5}$

Area = width · length

$3\,[3 + 3.5 + 9.5 + 4.5]$

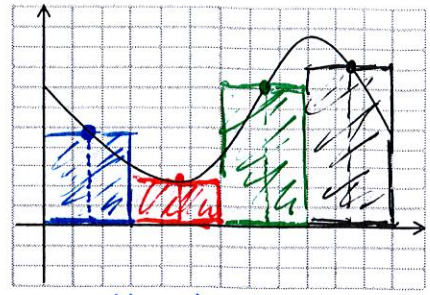
B. L_8



width = $\frac{12-0}{8} = 1.5$ $\boxed{A \approx 67.815}$

$A = [1.5] \cdot [7 + 5 + 3 + 2.25 + 3.5 + 7 + 9.5 + 8]$

C. M_4



width = $\frac{12-0}{4} = 3$

$A = 3\,[4.5 + 2.25 + 7 + 8]$

$\boxed{A = 65.25}$

Example 2: Estimate the area given the table. (These answers may not be different)

Compute R_6, L_6, & M_3 to estimate the distance traveled over the [0,3] if the velocity at half second intervals is as follows.

x t(s)	0	.5	1	1.5	2	2.5	3
y v(ft./s)	0	5	15	20	15	10	5

A. R_6

width = $\frac{3-0}{6} = .5$

Area = width · length

$= .5\,[5 + 15 + 20 + 15 + 10 + 5]$

$\boxed{A = 35}$

B. L_6

$.5\,[0 + 5 + 15 + 20 + 15 + 10]$

$\boxed{A = 32.5}$

C. M_3

width = $\frac{3-0}{3} = 1$

Area = width · length

$1\,[5 + 20 + 10]$

$\boxed{A = 35}$

Example 3: Estimate the area given the function. (These answers may not be different)

Let $f(x) = -x^2 + 4$, [0, 2]

A. R_4 width $= \frac{2-0}{4} = .5$

A= width · length

$A = .5\left[f(.5) + f(1) + f(1.5) + f(2)\right]$

$.5\left[3.75 + 3 + 1.75 + 0\right]$

$= \boxed{4.25}$

B. L_4

$A = .5\left[f(0) + f(.5) + f(1) + f(1.5)\right]$

$\boxed{6.25}$

C. M_2 width $= \frac{2-0}{2} = 1$

$A = 1\left[f(.5) + f(1.5)\right]$

$\boxed{5.5}$

Example 4: Estimate the area given the function. (These answers may not be different)

Let $f(x) = \ln x$, [1, 2] width $= \frac{2-1}{6} = \frac{1}{6}\left(\frac{1}{2}\right) = \frac{1}{12}$

M_6

$A = \frac{1}{6}\left[f(1\tfrac{1}{12}) + f(1\tfrac{3}{12}) + f(1\tfrac{5}{12}) + f(1\tfrac{7}{12}) + f(1\tfrac{9}{12}) + f(1\tfrac{11}{12})\right]$

$\frac{1}{6}\left[f\left(\frac{13}{12}\right) + f\left(\frac{15}{12}\right) + f\left(\frac{17}{12}\right) + f\left(\frac{19}{12}\right) + f\left(\frac{21}{12}\right) + f\left(\frac{23}{12}\right)\right]$

$\frac{1}{6}\left[.08 + .2231 + .3483 + .4595 + .5596 + .6506\right]$

$\frac{1}{6}\left[2.3211\right] \approx \boxed{.387}$

Example 4: Estimate the area given the function. (These answers may not be different)

Let $f(x) = \sin x$, $[0, \pi]$ width $= \frac{\pi - 0}{4} = \frac{\pi}{4}$

R_4

A= width · length

$\frac{\pi}{4}\left[f\left(\frac{\pi}{4}\right) + f\left(\frac{2\pi}{4}\right) + f\left(\frac{3\pi}{4}\right) + f\left(\frac{4\pi}{4}\right)\right]$

$\frac{\pi}{4}\left[.7071 + 1 + .7071 + 0\right]$