Notes: Integration By Parts Part I

Integration by Parts is a technique used for product-form integrands when u –Substitution fails.

Integration by Parts: $\int u dv = uv - \int v du$ $\quad$ (first) · (second)

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad u \cdot v$

Proof of formula:

$$\cancel{\int}\cancel{\frac{d}{dx}}[u\cdot v] = \int u\cdot dv + v\,du$$

$$u\cdot v = \int u\,dv + \cancel{\int v\,du}$$

$$-\int v\,du \qquad\qquad -\cancel{\int v\,du}$$

$$\int u\,dv = u\cdot v - \int v\,du$$

The most difficult part is picking your "u".

I-inverse trigonometric
L-logarithmic
A-algebraic (i.e. power functions)
T-trigonometric
E-exponential

The function that is highest is the acrostic will be the best choice for "u"

Integrate each:

Example One: $\int \underline{x}\underline{cosx\ dx}$

$u \cdot v - \int v\,du$

$x\cdot \sin x - \int \sin x\,dx$

$x\sin x - (-\cos x) + C$

$x\sin x + \cos x + C$

$\underline{u = x}$ $\qquad$ $v = \sin x$

$du = dx$ $\qquad$ $\underline{dv = \cos x\,dx}$

Example Two: $\int \underline{x}\underline{e^x\ dx}$

$u\cdot v - \int v\,du$

$x\cdot e^x - \int e^x\,dx$

$xe^x - e^x + C$

$\underline{u = x}$ $\qquad$ $v = e^x$

$du = dx$ $\qquad$ $\underline{dv = e^x\,dx}$

Example Three: $\int_1^3 \underline{lnx}\ \underline{dx}$

I-inverse trigonometric
L-logarithmic
A-algebraic
T-trigonometric
E-exponential

$\underline{u = \ln x}$ $\quad v = x$

$du = \frac{1}{x}dx \quad \underline{dv = dx}$

$u \cdot v - \int v\,du$

$(\ln x)x - \int \cancel{x}\left(\cancel{\tfrac{1}{x}}\right)dx$

$x\ln x - \int dx$

$x\ln x - x\Big]_1^3$

$3\ln(3) - 3 - (1)\ln(1) + 1$

$3\ln(3) - 2$

Example Four: $\int \underline{x^2} cosx\ dx$ (will show you a short cut tomorrow for this one ☺)

$\underline{u = x^2} \quad v = \sin x$

$du = 2x\,dx \quad \underline{dv = \cos x\,dx}$

$x^2\sin x - \int \underline{2x}\sin x\,dx$

$\underline{u = 2x} \quad v = -\cos x$

$du = 2dx \quad \underline{dv = \sin x\,dx}$

$x^2\sin x - [-2x\cos x - \int -2\cos x\,dx$

$x^2\sin x + 2x\cos x - \int 2\cos x\,dx$

$x^2\sin x + 2x\cos x - 2\sin x + C$

Sometimes You have to integrate by parts several times

Example Five: $\boxed{\int \underline{e^x} \underline{cosx}\ \underline{dx}}$

$\underline{u = \cos x} \quad v = e^x$

$du = -\sin x\,dx \quad \underline{dv = e^x dx}$

$\underline{u = \sin x} \quad v = e^x$

$du = \cos x\,dx \quad \underline{dv = e^x dx}$

$e^x\cos x - \int -e^x\sin x\,dx$

$e^x\cos x + \int \underline{e^x}\underline{\sin x}\,\underline{dx}$

$e^x\cos x + e^x\sin x - \boxed{\int e^x\cos x\,dx}$

Looks like You are going in circles

$\int e^x\cos x\,dx = e^x\cos x + e^x\sin x - \cancel{\int e^x\cos x\,dx}$

$+\int e^x\cos x\,dx \qquad\qquad +\cancel{\int e^x\cos x\,dx}$

$\cancel{\tfrac{1}{2}}\left(\cancel{2}\int e^x\cos x\,dx\right) = \cancel{\tfrac{1}{2}}\left(e^x\cos x + e^x\sin x\right) \quad \int e^x\cos x\,dx = \frac{1}{2}\left(e^x\cos x + e^x\sin x\right)$

Example Six: $\int \frac{\ln(lnx)}{x} dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$\int \ln u\,du$

Change to something other than u

$\int \underline{\ln x}\,\underline{dx}$

$\underline{u = \ln x} \quad v = x$

$du = \frac{1}{x}dx \quad \underline{dv = dx}$

$x\ln x - \int \cancel{x}\left(\cancel{\tfrac{1}{x}}\right)dx$

$x\ln x - \int dx = x\ln x - x + C$

Change Back to u:

$u\ln u - u + C$

Plug u - back in

$\ln x \cdot \ln(\ln x) - \ln x + C$

u-Substitution & then Integration By Parts