When: $h=10$ $A=100$ $A=\frac{1}{2}b\cdot h$ $100=\frac{1}{2}b(10)$ $100=5b$ $b=20$

Related Rate Organization

For most related rate problems the following thought process should be helpful.

Formula Problem: A particle moves along the curve $y=\sqrt{1+x^3}$. As it reaches the point (2, 3), the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x coordinate changing at that instant?

Know: $\frac{dy}{dt}=\frac{4cm}{sec}$

Find: $\frac{dx}{dt}=$ ___

When: $(2,3)$

Equation $\left[y=(1+x^3)^{1/2}\right]\frac{d}{dt}$

$(1)\frac{dy}{dt}=\frac{1}{2}(1+x^3)^{-1/2}(3x^2)\frac{dx}{dt}$

$\frac{dy}{dt}=\frac{3x^2}{2\sqrt{1+x^3}}\frac{dx}{dt}$

Derivative $\frac{4cm}{sec}=\frac{3(2)^2}{2\sqrt{1+(2)^3}}\frac{dx}{dt}$

Substitution $\frac{4cm}{sec}=\frac{12}{6}\frac{dx}{dt}$

$\frac{1}{2}\frac{4cm}{sec}=\cancel{2}\frac{dx}{dt}\cancel{\frac{1}{2}}$

$\frac{2cm}{sec}=\frac{dx}{dt}$

Formula Problem: The height of a triangle is increasing at a rate of 1cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the height is 10 cm and the area is $100cm^2$.

Know: $\frac{dh}{dt}=\frac{1cm}{min}$ & $\frac{dA}{dt}=\frac{2cm^2}{min}$

Find: $\frac{db}{dt}=$ ___ $b=20cm$

When: $h=10cm$ & $A=100cm^2$

Equation $\left[A=\frac{1}{2}b\cdot h\right]\frac{d}{dt}$

$(1)\frac{dA}{dt}=\frac{1}{2}b(1)\frac{dh}{dt}+h\left(\frac{1}{2}\right)\frac{db}{dt}$

Derivative $\frac{dA}{dt}=\frac{1}{2}b\frac{dh}{dt}+\frac{1}{2}h\frac{db}{dt}$

$\frac{2cm^2}{min}=\frac{1}{2}(20cm)\frac{1cm}{min}+\frac{1}{2}(10cm)\frac{db}{dt}$

Substitution $\frac{2cm^2}{min}=\frac{10cm^2}{min}+5cm\frac{db}{dt}$

$-\frac{10cm^2}{min}$ $\cancel{-\frac{10cm^2}{min}}$

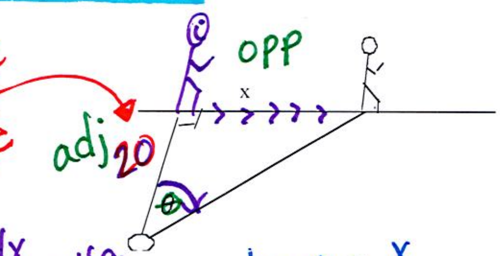
$\left(\frac{1}{5cm}\right)\frac{-8cm^2}{min}=5cm\frac{db}{dt}\left(\frac{1}{5cm}\right)$

$\frac{db}{dt}=\frac{-8}{5}\frac{cm}{min}$

Soh cah toa

Angle Problem: A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?

Because does not change



Know: $\frac{dx}{dt}=\frac{4ft}{sec}$

Find: $\frac{d\theta}{dt}$

When: $x=15$

$\tan\theta=\frac{x}{20}$

$\tan\theta=\frac{15}{20}$

$\theta=\tan^{-1}(15/20)=.644$

Equation $\tan\theta=\frac{opp}{adj}$ $\tan\theta=\frac{x}{20}$

$\frac{d}{dt}\left[20\tan\theta=x\right]$

Derivative $\frac{\cancel{20\sec^2\theta}\cdot\frac{d\theta}{dt}}{\cancel{20\sec^2\theta}}=\frac{(1)\frac{dx}{dt}}{20\sec^2\theta}$

Substitution $\frac{d\theta}{dt}=\frac{\cos^2\theta}{20}\frac{dx}{dt}$

$\frac{d\theta}{dt}=\frac{[\cos(.644)]^2}{20\cancel{ft}}\frac{4\cancel{ft}}{sec}=.128\frac{Rad}{s}$

Are always Rates: $\frac{d(\text{something})}{dt}$

Related Rate Organization

For most related rate problems the following thought process should be helpful. The only Reason you can plug the number 20 into the equation before you take a derivative is because the length of the ladder does ~~not~~ not change in this problem.

1. Read the problem.
2. Answer these questions

Know: What rate are you given? (ie. $\frac{dv}{dt}$)

Find: What rate are you asked to find?

When: Most of the time you are given a time when you want to find the rate. *This information is only used after the derivative is taken. If this information is not given, then you will not need it.*

3. Write an equation that relates the **Know** rate with the **Find** rate.

   You may only use the variables from the **Know** and the Find. If other variables are present in the equation you must look for additional information. You will use this information to replace the unwanted variable.

4. Take a derivative. Since we are concerned about WHEN things occur we will be taking derivative with respect to time (t) and we will need to do implicit differentiation.

   $\frac{d}{dt}[\text{Equation}]$

5. Substitute in the known rate and the when time.

6. Evaluate and label your answer.

Sphere problem: Joe inflates a spherical balloon. Air is entering the balloon at a rate of 15 $\frac{cm^3}{sec}$. How fast is the radius changing when the radius is 10 cm.

Know: $\frac{dV}{dt} = \frac{15cm^3}{sec}$

Find: $\frac{dR}{dt} = \_\_\_$

When: $R = 10cm$

Equation $\left[V = \frac{4}{3}\pi R^3\right]\frac{d}{dt}$

$(1)\frac{dV}{dt} = \cancel{\frac{4}{3}}\pi(\cancel{3}R^2)\frac{dR}{dt}$

Derivative

$\frac{dV}{dt} = 4\pi R^2\frac{dR}{dt}$

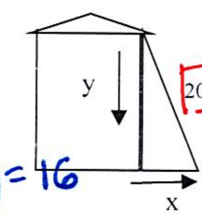
$\frac{15cm^3}{sec} = 4\pi(10cm)^2\frac{dR}{dt}$

Substitution

$\frac{1}{400\pi \cancel{cm^2}} \cdot \frac{15\cancel{cm^3}}{sec} = \cancel{400\pi cm^2}\frac{dR}{dt} \cdot \frac{1}{\cancel{400\pi cm^2}}$

$\frac{dR}{dt} = \frac{3}{80\pi}\frac{cm}{sec}$

Ladder Problem: A 20 foot ladder is leaning against a house. The foot of the ladder begins to slide away from the house at a rate of 2 feet/second. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 12 feet from the house?



Know: $\frac{dx}{dt} = \frac{2ft}{sec}$

Find: $\frac{dy}{dt} = \_\_\_$

When: $x = 12ft$ $y = 16$

Equation $x^2 + y^2 = 20^2$

$\frac{d}{dt}\left[x^2 + y^2 = \boxed{400}\right]$

Derivative

$\frac{2x\frac{dx}{dt}}{2} + \frac{2y\frac{dy}{dt}}{2} = \frac{0}{2}$

$x^2 + y^2 = 400$

$12^2 + y^2 = 400$

$y^2 = 256$

$y = 16$

Substitution $x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$(12ft)\left(\frac{2ft}{sec}\right) + (16ft)\frac{dy}{dt} = 0$

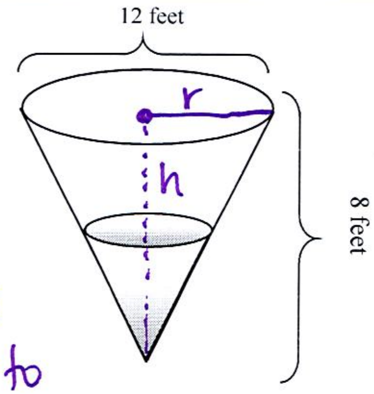
$\frac{1}{\cancel{16ft}}\cancel{16ft}\frac{dy}{dt} = -\frac{24ft^2}{sec}\frac{1}{16ft}$

$\frac{dy}{dt} = \frac{-3}{2}\frac{ft}{sec}$

Related Rate Organization

For most related rate problems the following thought process should be helpful.

Cone Problem: A water tank is in the shape of an inverted cone with a diameter of 12 feet and a depth of 8 feet.



If the water is 3 feet deep, and is rising at a rate of 2 feet/hr, at what rate is the volume changing?

Know: $\frac{dh}{dt} = \frac{2ft}{hR}$

Find: $\frac{dV}{dt} =$

When: $h = 3ft$

I want to get Rid of R

$\frac{r}{h} = \frac{6}{8}$ $\quad \frac{\cancel{8}r}{\cancel{8}} = \frac{6h}{8}$ $\quad r = \frac{3}{4}h$

Equation $V = \frac{1}{3}\pi R^2 h$

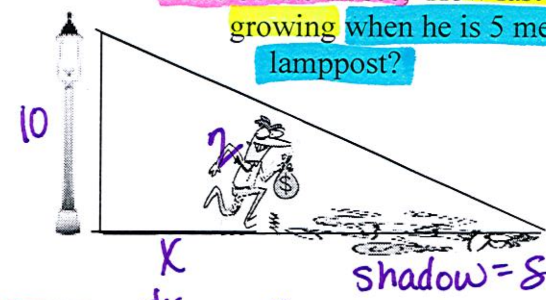
$V = \frac{1}{3}\pi\left(\frac{3}{4}h\right)^2 h = \frac{1}{\cancel{3}}\pi\left(\frac{\cancel{9}^{3}}{16}h^2\right)h = \frac{3}{16}\pi h^3$

Derivative $\frac{d}{dt}\left[V = \frac{3}{16}\pi h^3\right]$ $\quad \frac{dV}{dt} = \frac{3}{16}\pi(3h^2)\frac{dh}{dt}$

Substitution $\frac{dV}{dt} = \frac{9}{16}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{9}{16}\pi(3ft)^2\left(\frac{2ft}{h}\right) = \frac{9}{8\cancel{16}}\pi(9ft^2)\left(\frac{\cancel{2}^{1}ft}{h}\right) = \frac{81\pi}{8}\frac{ft^3}{h}$

Shadow Problem: A pickpocket walking away from a 10 meter tall lamppost is 2 meters tall. He walks at a rate of 1.5 m/sec, How fast is his shadow growing when he is 5 meters from the lamppost?



Know: $\frac{dx}{dt} = 1.5\frac{m}{s}$

Find: $\frac{ds}{dt} =$ ___

When: $x = 5m$

Equation $\frac{s}{2} = \frac{s+x}{10}$ $\quad 10s = \cancel{2s} + 2x$ $\quad -2s$

$\frac{d}{dt}[8s = 2x]$

Derivative $\frac{8ds}{dt} = 2\frac{dx}{dt}$ $\quad \frac{1}{\cancel{4}}\frac{\cancel{4}ds}{dt} = \frac{dx}{dt}\left(\frac{1}{4}\right)$

Substitution $\frac{ds}{dt} = \frac{1}{4}\frac{dx}{dt}$ $\quad \frac{ds}{dt} = \frac{1}{4}\left(1.5\frac{m}{s}\right) = .375\frac{m}{s}$

How fast is the tip of the shadow moving when he is 5 meters from the lamppost?

tip of shadow $= \frac{dx}{dt} + \frac{ds}{dt} = 1.5\frac{m}{s} + .375\frac{m}{s} = 1.875\frac{m}{s}$

Notes: Related Rates (1) length = initial + Rate · time

Related Rates Problems: Are word problems where something is changing over time. To solve the problems you find the same things over and over again. Know, Find, When, Equation, Derivative(you are always taking a derivative with respect to time), and finally you Substitute in. These same 6 steps over and over again.

1. A stone is thrown into a pond creating ripples that are concentric circles. The rate of change of the radius of the circle is 2cm/sec. Find the rate of change of the area of the circle when the radius is 12cm.

Know: $\frac{dR}{dt} = 2\frac{cm}{sec}$

Find: $\frac{dA}{dt} = \_\_\_$

When: $R = 12cm$

Equation: $\left[A = \pi R^2\right]\frac{d}{dt}$

Derivative:

$(1)\frac{dA}{dt} = \pi(2R)\frac{dR}{dt}$

$\frac{dA}{dt} = 2\pi R\frac{dR}{dt}$

Substitution:

$\frac{dA}{dt} = 2\pi(12cm)\left(\frac{2cm}{sec}\right)$

$\frac{dA}{dt} = 48\pi\frac{cm^2}{sec}$

2. Ivy throws a snowball at her mom Mrs. MacIntyre. The snowball is melting at a rate of $\frac{1}{2}\frac{cm^3}{\min}$. At what rate is the radius of the snowball melting when the radius is $20cm$? $V_{sphere} = \frac{4}{3}\pi r^3$

Know: $\frac{dV}{dt} = -\frac{1}{2}\frac{cm^3}{min}$

Find: $\frac{dR}{dt} = \_\_\_$

When: $R = 20cm$

Equation: $\left[V = \frac{4}{3}\pi R^3\right]\frac{d}{dt}$

Derivative:

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi(\cancel{3}R^2)\frac{dR}{dt}$

Substitution:

$\frac{dV}{dt} = 4\pi R^2\frac{dR}{dt}$

$-\frac{1}{2}\frac{cm^3}{min} = 4\pi(20cm)^2\frac{dR}{dt}$

$\frac{1}{1600\pi cm^2}\cdot -\frac{1}{2}\frac{cm^3}{min} = \cancel{1600\pi cm^2}\frac{dR}{dt}\cdot\frac{1}{\cancel{1600\pi cm^2}}$

$\frac{dR}{dt} = \frac{-1}{[illegible]\pi}\frac{cm}{min}$

3. A stone is thrown into a pond creating ripples that are concentric circles. The rate of change of the radius of the circle is 3 ft/min. If the radius is 0 at time=0, how fast is the area increasing after 4 minutes?

Know: $\frac{dR}{dt} = \frac{3ft}{min}$

Find: $\frac{dA}{dt} = \_\_\_$

When: $t = 4min.$

$R = 0 + \frac{3ft}{\cancel{min}}(4\cancel{min}) = 12ft$

Equation: $\left[A = \pi R^2\right]\frac{d}{dt}$

Derivative: $\frac{dA}{dt} = \pi(2R)\frac{dR}{dt}$

$\frac{dA}{dt} = 2\pi R\frac{dR}{dt}$

Substitution:

$\frac{dA}{dt} = 2\pi(12ft)\left(\frac{3ft}{min}\right)$

$\frac{dA}{dt} = 72\pi\frac{ft^2}{min}$

4. A 20 foot ladder is leaning against a house. The foot of the ladder begins to slide away from the house at a rate of 2 feet/second. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 16 feet from the house?

Know: $\frac{dx}{dt} = \frac{2ft}{sec}$

Find: $\frac{dy}{dt} = \_\_\_$

When: $y = 12ft$

$x = 16ft$

$16^2 + y^2 = 20^2$

$y^2 = 144$

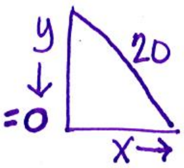
$y = 12$

Equation: $\left[x^2 + y^2 = 20^2\right]\frac{d}{dt}$

Derivative: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

Substitution



$(16ft)\left(\frac{2ft}{sec}\right) + (12ft)\frac{dy}{dt} = 0$

$\left(\frac{1}{-12ft}\right)\frac{32ft^2}{sec} = \cancel{-12ft}\frac{dy}{dt}\left(\frac{1}{\cancel{-12ft}}\right)$

$\frac{dy}{dt} = \frac{-8}{3}\frac{ft}{sec}$

5. An 18 foot ladder is leaning against a house. The top of the ladder begins to slide down the house at a rate of 3 feet/second. How fast is the ladder sliding away from the wall after 2 minutes. The ladder is 4 feet from the wall at time=0.

Know: Equation: Substitution:

Find: Derivative:

When:

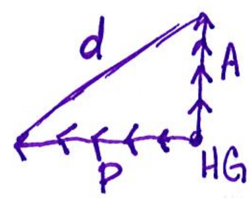
omit

6. Adrianna and Paul are on two four-wheelers located in the parking lot of Hillgrove High school. At time=0 Adrianna travels north at a speed of 30mph. Paul travels west at a speed of 35mph.

a. How far has Adrianna and Paul traveled after 20 minutes?

$\frac{20min}{60min} = \frac{1}{3}hR$

$A = \frac{30mi}{\cancel{hR}}\left(\frac{1}{3}\cancel{hR}\right) = 10mi \quad P = \frac{35mi}{\cancel{hR}}\left(\frac{1}{3}\cancel{hR}\right) = 11.\bar{6}\ mi$



b. At what rate is the distance between them increasing at time=20 minutes?

Know: $\frac{dA}{dt} = \frac{30mi}{hR}$ $\frac{dP}{dt} = \frac{35mi}{hR}$

Find: $\frac{dd}{dt} =$ ___

When:

$t = 20min$

$A = 10mi$

$P = 11.\bar{6}\ mi$

$A^2 + P^2 = d^2$

$\sqrt{10^2 + 11.\bar{6}^2} = d$

$d = 15.366\ mi$

Equation: $\frac{d}{dt}\left[A^2 + P^2 = d^2\right]$

Derivative:

$2A\frac{dA}{dt} + 2P\frac{dP}{dt} = 2d\frac{dd}{dt}$

$A\frac{dA}{dt} + P\frac{dP}{dt} = d\frac{dd}{dt}$

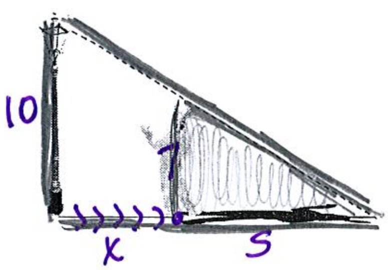
Substitution:

$10mi\left(\frac{30mi}{hr}\right) + (11.\bar{6}mi)\left(\frac{35mi}{hr}\right) = (15.366\ mi)\frac{dd}{dt}$

$\frac{1}{15.366mi} \quad \frac{708.\bar{3}\ mi^2}{hr} = \cancel{15.366mi}\frac{dd}{dt} \quad \frac{1}{\cancel{15.366mi}}$

$\frac{dd}{dt} = 46.097\frac{mi}{hR}$

7. A man in a 7 foot tall bunny costume is walking away from a 10 foot tall street lamp at a rate of 3 feet per second. How fast is the man's showdown growing?



Know: $\frac{dx}{dt} = \frac{3ft}{s}$

Find: $\frac{ds}{dt}$

When:

Equation: $\frac{s}{7} = \frac{x+s}{10}$

Derivative: $10s = 7x + \cancel{7s}$
$-7s \quad -\cancel{7s}$

$\frac{d}{dt}\left[3s = 7x\right]$

$3\frac{ds}{dt} = 7\frac{dx}{dt}$

Substitution:

$\frac{\cancel{3}\frac{ds}{dt}}{\cancel{3}} = \frac{7\left(\cancel{3}\frac{ft}{s}\right)}{\cancel{3}}$

$\frac{ds}{dt} = \frac{7ft}{sec}$