

Notes: Formal Definition of Derivative

Terminology and Notation:

The *derivative* of f at $x=a$ is the instantaneous rate of change of f at $x=a$. The following notations are used for the derivative of f at $x=a$:

$$f'(a) \qquad \left. \frac{df}{dx} \right|_{x=a}$$

Derivative: $f'(x)$	Derivative at a point: $f'(a)$ where a is some number	
$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$	$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$	$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

Example(s) 1:

A.) Find $\frac{dy}{dx}$ for $f(x) = x^2 + 1$

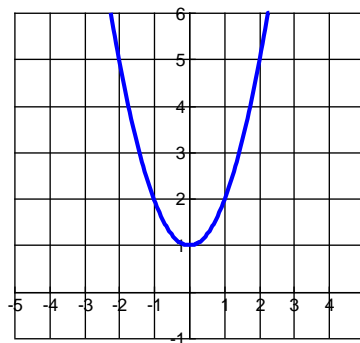
B.) Find $\left. \frac{dy}{dx} \right|_{x=-1}$

C.) Find $\left. \frac{dy}{dx} \right|_{x=0}$

D.) Find $\left. \frac{dy}{dx} \right|_{x=1}$

E.) Find $\left. \frac{dy}{dx} \right|_{x=2}$

What does this mean graphically?



Example 2:

Find $f'(x)$ for $f(x) = \sqrt{x-2}$

Example 3:

Remember: $f'(x)$ is the same as instant rate of change.

Find $f'(x)$ for $f(x) = x^3 - 2x$

Example 4:

Find the instant rate of change of the function at $x = 2$

$$f(t) = 6 + \sqrt{t}$$

Tangent Lines: You will always need two things for a tangent line:

1. Point
2. Slope

There are three forms of a line:

1. Slope / Intercept
2. Point/Slope
3. General

Example 5:

Find the equation for the tangent line to $g(x) = \frac{8}{x}$ at $x = 4$.

Example 6:

Find the equation for the tangent line to $g(x) = 2x - 3$ at $x = 2$.

Memorize each of these!!!

Derivative: $f'(x)$	Derivative at a point: $f'(a)$ where a is some number	
$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$	$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$	$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

Example(s) 7:

What is $f(x)$ in each of the following? What is each question asking you to answer?

$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$	$\lim_{h \rightarrow 0} \frac{\sqrt{4+h} - \sqrt{4}}{h}$ or $\lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h}$	$\lim_{x \rightarrow 4} \frac{\sqrt{x} - \sqrt{4}}{x - 4}$ or $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$
$\lim_{h \rightarrow 0} \frac{(x+h)^5 - x^5}{h}$	$\lim_{h \rightarrow 0} \frac{(2+h)^5 - 32}{h}$	$\lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2}$
$\lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h}$	$\lim_{h \rightarrow 0} \frac{\cos(\pi+h) - (-1)}{h}$	$\lim_{x \rightarrow \pi} \frac{\cos(x) - (-1)}{x - \pi}$
$\lim_{x \rightarrow -1} \frac{(x^2 + 2) - (3)}{x + 1}$	$\lim_{h \rightarrow 0} \frac{(x+h)^2 + 2 - (x^2 + 2)}{h}$	$\lim_{h \rightarrow 0} \frac{(-1+h)^2 + 2 - (3)}{h}$
$\lim_{h \rightarrow 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$	$\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$	$\lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$
$\lim_{h \rightarrow 0} \frac{[(x+h)^3 - 2] - (x^3 - 2)}{h}$	$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$	$\lim_{h \rightarrow 0} \frac{\ln(5+h) - \ln 5}{h}$
$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sec x - 1}{x - \frac{\pi}{2}}$	$\lim_{h \rightarrow 0} \frac{e^{2+h} - e^2}{h}$	$\lim_{h \rightarrow 0} \frac{[x+h - 1] - (x - 1)}{h}$