P19

What is the Formula for Lagrange Error bound? How do you find K?

Formula:

max Error $= \frac{K}{(n+1)!}|x-a|^{n+1}$

To Find K:

$\boxed{1.}$ Find $f^{n+1}(a)$ find the n+1 derivative

$\boxed{2.}$ Find $|f^{n+1}(x)|$ & Find $|f^{n+1}(a)|$

$\boxed{3.}$ The larger value from step 2 = K.

P20

Find the max error for $|T_3(1.5)-\sqrt{1.5}|$ centered at c=1

Max Error

$\frac{K}{(n+1)!}|x-a|^{n+1}$

$\frac{.9375}{4!}|1.5-1|^4$

$=\frac{.9375}{4!}(.5)^4$

$=.002441$

$T_3$ for $f(x)=\sqrt{x}$

$\boxed{1}$ Need $f^4(x)$

$f(x)=x^{1/2}$

$f'(x)=\frac{1}{2}x^{-1/2}$

$f''(x)=-\frac{1}{4}x^{-3/2}$

$f^3(x)=\frac{3}{8}x^{-5/2}$

$f^4(x)=-\frac{15}{16}x^{-7/2}$

$\boxed{2}$ $f^4(1)=\frac{-15}{16(\sqrt{1})^7}$ $=15/16$ $=.9375$

$f^4(1.5)=\frac{-15}{16(\sqrt{1.5})^7}$ $=.226$

$\boxed{3}$ $K=.9375$

P21

A Taylor series centered about x=5 has an $n^{th}$ derivative $f^n(5)=\frac{(-1)^n n!}{2^n(n+2)}$, Show that the 6th degree Taylor approximates f(6) with error less than $\frac{1}{1000}$

max Error $<\frac{1}{1000}$ so $\frac{K}{(n+1)!}|x-a|^{n+1}<\frac{1}{1000}$

$\frac{K}{7!}|x-a|^7<\frac{1}{1000}$ 6th degree

$\frac{K}{7!}|6-5|^7<\frac{1}{1000}$ find f(6) centered at 5

K = bigger of $|f^7(6)|$ OR $|f^7(5)|$

$\frac{\cancel{7!}}{2^7(7+2)\cancel{7!}}(1)^7<\frac{1}{1000}$ All you have $f^n(5)=\frac{(-1)^n n!}{2^7(n+2)}$ Use $|f^7(5)|$

$\frac{1}{2^7(9)}<\frac{1}{1000}$ $\frac{1}{128(9)}<\frac{1}{1000}$ $\frac{1}{1152}<\frac{1}{1000}$

☺ True max error $<\frac{1}{1000}$