

PPV 3

If you have
a parametric
equation

$$C(t) = (x(t), y(t))$$

OR

$$x(t) = _ \text{ \& } y(t) = _$$

Then $\frac{dy}{dx} =$ And $\frac{d^2y}{dx^2} =$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\text{derivative of } y}{\text{derivative of } x}$$

$$\frac{d^2y}{dx^2} = \frac{x'(t) \cdot y''(t) - y'(t) \cdot x''(t)}{[x'(t)]^3}$$

Find the equation
of the tangent
line to

$$x = t+1, y = t^2+3t$$

at the point (0, -2)

PPV 4

Tangent line

1. Point (0, -2)

2. Slope at $t = -1$

Solve for t

$$0 = t+1 \text{ \& } -2 = t^2+3t$$

$$t^2+3t+2=0$$

$$(t+1)(t+2)=0$$

$$t = -1, -2$$

Given $x = t+1, y = t^2+3t$

$$\frac{dy}{dx} = \frac{2t+3}{1} = \frac{2(-1)+3}{1} = 1$$

$$y - (-2) = 1(x - 0) \quad \text{y+2 = 1(x-0)}$$

Find the
intervals of
concavity for
 $x = t^2, y = \ln t$

PPV 5

$$x = t^2, y = \ln t$$

$$x' = 2t, y' = \frac{1}{t}$$

$$x'' = 2, y'' = -\frac{1}{t^2}$$

$$\frac{d^2y}{dx^2} = \frac{2t(-\frac{1}{t^2}) - \frac{1}{t}(2)}{[2t]^3} = \frac{-\frac{2}{t} - \frac{2}{t}}{8t^3} = \frac{-4}{8t^3} = -\frac{1}{2t^3}$$

$\frac{d^2y}{dx^2} = -\frac{1}{2t^3}$ ← always neg
 $\frac{d^2y}{dx^2} = \frac{1}{2t^3}$ ← always pos
 Concave down: $(-\infty, \infty)$

How do you find the vertical & horizontal tangents of a parametric equation? PPV6

Vertical tangent: set bottom of derivative = 0 & solve

$$\frac{dx}{dt} = 0$$

Horizontal tangent: set top of derivative = 0 & solve

$$\frac{dy}{dt} = 0$$

Find the point(s) where $x = t^2 - t + 2$, $y = t^3 - 3t$ has vertical tangents. PPV
7

$$x = t^2 - t + 2 \quad y = t^3 - 3t$$

Vertical tangent $x'(t) = 0$
 $2t - 1 = 0$
 $t = \frac{1}{2}$

$$\begin{aligned} \text{Point: } x &= \left(\frac{1}{2}\right)^2 - \frac{1}{2} + 2 & y &= \left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right) \\ x &= \frac{1}{4} - \frac{1 \cdot 2}{2 \cdot 2} + \frac{2 \cdot 4}{4} & y &= \frac{1}{8} - \frac{3 \cdot 4}{2 \cdot 4} \\ x &= \frac{1}{4} - \frac{2}{4} + \frac{8}{4} & y &= \frac{1}{8} - \frac{12}{8} \\ & \left(\frac{7}{4}, -\frac{11}{8}\right) \end{aligned}$$