What are the 3 conditions that must be met for the function f(x) to be continuous at x=a?

1. $f(a)$ is defined (no hole)

2. $\lim_{x \to a} f(x)$ exists

3. $f(a) = \lim_{x \to a} f(x)$

What types of functions have Removable verses non-Removable discontinuities?

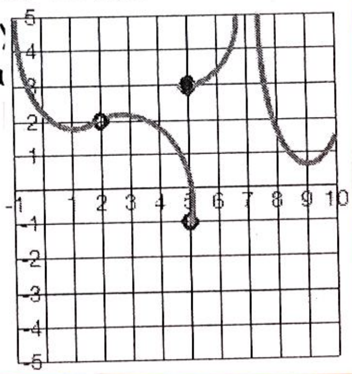
Removable vs non Removable

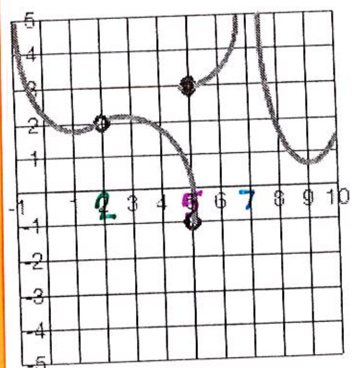
hole vs where the limit does not exist

→ gaps

→ vertical asymptotes

Look at the graph. State where the graph is discontinuous and why it is discontinuous. Is the discontinuity non- remova





X= 2

Because f(2) is undefined

hole ☺

Removeable/Nonremoveable

X= 5

Because $\lim_{x \to 5} f(x) =$ does not exist

Removeable/Nonremoveable gap ☺

X= 7

Because f(7) is undefined

Removeable/Nonremoveable

Asymptote ☺

Show that

$$f(x)=\begin{cases} x^2+3, & x>-1 \\ 2x+6, & x<-1 \\ 2, & x=-1 \end{cases}$$

is continuous everywhere or state where $f(x)$ is discontinuous and why.

$$f(x)=\begin{cases} x^2+3, & x>-1 & \text{Right} \\ 2x+6, & x<-1 & \text{left} \\ 2, & x=-1 & \text{function value} \end{cases}$$

Continuous $(-\infty,-1) \cup (-1,\infty)$

$\lim_{x \to -1^+} f(x) = (-1)^2+3 = 4$

$\lim_{x \to -1^-} f(x) = 2(-1)+6 = 4$

$f(-1) = 2$

f(x) is discontinuous at x=-1 because $\lim_{x \to -1} f(x) \neq f(-1)$