

Please start off every review with reading your notecards for that unit several times!!!! This is a very limited review!!!!

Limit-intended height (y-value) of the functions.

Properties- add, subtract, multiply, divide, multiply constant & raise to power.

Techniques to Evaluate-

- Direct Substitution-plug the x-value in... if you get a number you are done... If you get an indeterminate form...
 1. Try to factor the expression. Cancel common factors & direct substitution.
 2. Try table or graphs... try plugging in a number close to the x-value on the right/left
 3. L'Hopital's Rule

One Sided limits-Right sided: $\lim_{x \rightarrow c^+} f(x)$ Left sided: $\lim_{x \rightarrow c^-} f(x)$ If $\lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^-} f(x)$ Then $\lim_{x \rightarrow c} f(x) = \text{exists}$

Infinite Limits- $\lim_{x \rightarrow \infty} f(x) = \text{EndBehavior}$ HA: $y = 0 / \#$ $\lim_{x \rightarrow c} \frac{\#}{x - c} = \pm\infty$ VA: $x = c$

- (Ladies) J-Lo=0 Marilyn=Leading Coeff Dolly = $\pm\infty$ (plug in #) • Plug in #

Practice: Non-Calculator Active-2008 (1-3) Calculator Active (4-5)

1. $\lim_{x \rightarrow \infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is
- 3
 - 2
 - 2
 - 3
 - nonexistent

2. $\lim_{x \rightarrow 0} \frac{5x^4 + 8x^2}{3x^4 - 16x^2}$ is
- 1/2
 - 0
 - 1
 - 5/3 + 1
 - Nonexistent

Key:
1.B
2.A
3.A
4.C
5.E

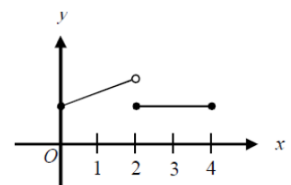
3. Let f be the function defined to the right. Which of the following statements about f are true?

- f has a limit at $x=2$.
 - f is continuous at $x=2$.
 - f is differentiable at $x=2$.
- I only
 - II only
 - III only
 - I & II only
 - I, II, & III

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

4. The figure to the right shows the graph of a function f with domain $0 \leq x \leq 4$. Which of the following statements are true?

- $\lim_{x \rightarrow 2^-} f(x)$ exists
 - $\lim_{x \rightarrow 2^+} f(x)$ exists
 - $\lim_{x \rightarrow 2} f(x)$ exists
- I only
 - II only
 - I & II only
 - I & III only
 - I, II, & III



5. The function f is continuous for $-2 \leq x \leq 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$ for which $f'(c) = 0$, which of the following statements must be true?

- For $-2 < k < 2$, $f'(k) > 0$.
- For $-2 < k < 2$, $f'(k) < 0$.
- For $-2 < k < 2$, $f'(k)$ exists.
- For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.
- For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

Practice: Non-Calculator Active-2003 (6-9) Calculator Active (10)

Key:
6. E
7. C
8. A
9. D
10.D

6. For $x \geq 0$, the horizontal line $y=2$ is an asymptote for the graph of the function f . Which of the following must be true?

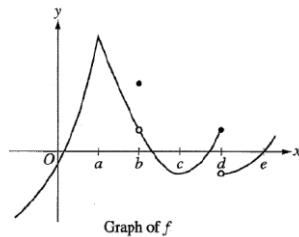
- a. $f(0)=2$
- b. $f(x) \neq 2$ for all $x \geq 0$
- c. $f(2)$ is undefined
- d. $\lim_{x \rightarrow 2} f(x) = \infty$
- e. $\lim_{x \rightarrow \infty} f(x) = 2$

7. $\lim_{x \rightarrow \infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

- a. 4
- b. 1
- c. 1/4
- d. 0
- e. -1

8. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

- a. a
- b. b
- c. c
- d. d
- e. e

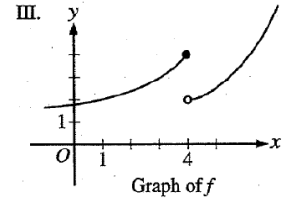
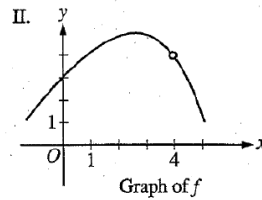
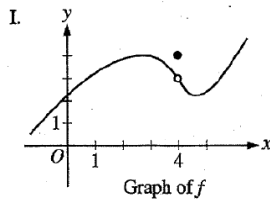


9. Let f be the function given to the right. Which of the following statements are true about f ?

- I. $\lim_{x \rightarrow 3} f(x)$ exists.
 - II. f is continuous at $x=3$.
 - III. f is differentiable at $x=3$.
- $f(x) = \begin{cases} x+2 & \text{if } x \leq 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$
- a. None
 - b. I only
 - c. II only
 - d. I & II only
 - e. I, II, & III

10. For which of the following does $\lim_{x \rightarrow 4} f(x)$ exist?

- a. I only
- b. II only
- c. III only
- d. I & II only
- e. I & III only



Free Response Practice

Free Response 2011 #6 **Non-Calculator**

11. Let f be a function defined by

$$f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0 \end{cases}$$

a.) Show that f is continuous at $x=0$.

Free Response 2011B #2 **Calculator**

12. A 12,000 liter tank of water is filled to capacity. At $t=0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-

defined function $r(t) = \begin{cases} \frac{600t}{t+3} & \text{for } 0 \leq t \leq 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$

a.) Is r continuous at $t=5$? Show the work that leads to your answer.

Free Response 2008 #6 **Non-Calculator**

13. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for

all $x > 0$. The derivative of f is given by

$$f'(x) = \frac{1 - \ln x}{x^2},$$

a.) Find $\lim_{x \rightarrow 0^+} f(x)$

Free Response 2003 #6 **Non-Calculator**

14. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5 \end{cases}.$$

a.) Is f continuous at $x=3$? Explain why or why not?

Free Response Practice **Non-Calculator**

15. Given the function $f(x) = \frac{x^3 + 2x^2 - 3x}{3x^2 + 3x - 6}$.

a.) What are the zeros of $f(x)$?

b.) What are the vertical asymptotes of $f(x)$?

c.) The end behavior model of $f(x)$ is the function $g(x)$. What is $g(x)$?

d.) What is $\lim_{x \rightarrow \infty} f(x)$? What is $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$?