Infinite Series

If you are asked to determine if an alternating series converges absolutely, converges conditionally or converges not at all... → What does that mean?

If you have $\sum_{n=1}^{\infty} (-1)^n a_n$ it will

$\boxed{\text{converge absolutely}}$ If $\sum_{n=1}^{\infty} a_n$ converges

$\boxed{\text{converge conditionally}}$ If $\sum_{n=1}^{\infty} a_n$ diverges <u>And</u> $\sum_{n=1}^{\infty} (-1) a_n$ converges

$\boxed{\text{converge NOT at All}}$ If $\sum_{n=1}^{\infty} a_n$ diverges <u>And</u> $\sum_{n=1}^{\infty} (-1) a_n$ diverges

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Determine if $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges

→ Absolutely

→ Conditionally

→ or Not at all

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

$\boxed{\text{Positive}}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ **converges** by p-series $p=2$ which is $p>1$.

so $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ **converges absolutely**!

Determine if $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges

→ absolutely

→ conditionally

→ or not at all

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$\boxed{\text{Positive}}$

$\sum_{n=1}^{\infty} \frac{1}{n}$

diverges by p-series $p=1$ so $p \leq 1$.

$\boxed{\text{Alternating}}$

$\sum_{n=1}^{\infty} \frac{(-1)}{n}$ $\frac{1}{n}$ is decreasing

$\lim_{n\to\infty} \frac{1}{n} = 0$

Converges by Alt. Series Test

so $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ **converges conditionally**

Infinite Series

Determine if

$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{4}{3}\right)^n$$

Converges
- → Absolutely
- → Conditionally
- → or not at all

$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{4}{3}\right)^n$$

Positive

$$\sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$$

diverges by geometric $R = \frac{4}{3}$ which is $|R| \geq 1$.

Alternating

$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{4}{3}\right)^n = \sum_{n=1}^{\infty} \left(\frac{-4}{3}\right)^n$$

diverges by geometric $r = -\frac{4}{3}$ which is $|R| \geq 1$.

$\sum_{n=1}^{\infty} (-1)^n \left(\frac{4}{3}\right)^n$ **converges not at all.**

Infinite Series

Complete the chart