Infinite Series

When do you use the geometric test?
When does it converge?
When does it diverge?
How do you find the sum?

If you have $\sum_{n=0}^{\infty} c(R)^n$

where R = common Ratio

converges: $|R| < 1$

diverges: $|R| \geq 1$

$$\text{Sum} = \frac{\text{first term}}{1 - \text{Ratio}}$$

Infinite Series

Does $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$ converge or diverge?

If it converges then find the sum.

$\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$ Converges because geometric $R = \frac{2}{3} < 1$.

$$\text{Sum} = \frac{5\left(\frac{2}{3}\right)^0}{1-\frac{2}{3}} = \frac{5}{\frac{3}{3}-\frac{2}{3}} = \frac{5\cdot\frac{3}{1}}{\frac{1}{3}\cdot\frac{3}{1}}$$

$$\text{Sum} = \boxed{15}$$

Infinite Series

Find the sum of the geometric Series

$$\sum_{n=2}^{\infty} 3^{2n}5^{-2n} + 2^{-n}$$

$$\sum_{n=2}^{\infty} 3^{2n}\cdot 5^{-2n} + 2^{-n} = \sum_{n=2}^{\infty} \frac{3^{2n}}{5^{2n}} + \sum_{n=2}^{\infty} \frac{1}{2^n}$$

$$= \sum_{n=2}^{\infty} \frac{(3^2)^n}{(5^2)^n} + \sum_{n=2}^{\infty} \left(\frac{1}{2}\right)^n = \sum_{n=2}^{\infty} \left(\frac{9}{25}\right)^n + \sum_{n=2}^{\infty} \left(\frac{1}{2}\right)^n$$

Both terms converge by geometric $R < 1$.

$a_2 = \frac{81}{625}$ $R = \frac{9}{25}$ $\qquad$ $a_2 = \frac{1}{4}$ $R = \frac{1}{2}$

$$\frac{\frac{81}{625}}{\frac{25}{25}\cdot 1 - \frac{9}{25}} = \frac{\frac{81}{625}\cdot\frac{25}{16}}{\frac{16}{25}\cdot\frac{25}{16}} = \frac{81}{400} \qquad \frac{\frac{1}{4}}{\frac{2}{2}\cdot 1 - \frac{1}{2}} = \frac{\frac{1}{4}\cdot\frac{2}{1}}{\frac{1}{2}\cdot\frac{2}{1}} = \frac{1}{2}$$

$$\frac{81}{400} + \frac{1(200)}{2(200)} = \frac{81}{400} + \frac{200}{400} = \frac{281}{400}$$

Infinite series

What is the divergence test?

If you have $\sum_{n=0}^{\infty} a_n$

diverges: If $\lim_{n\to\infty} a_n \neq 0$

converges: You can not tell converges with this test.

If $\lim_{n\to\infty} a_n = 0$ then you must try another test.

How can $\lim_{n\to\infty} \frac{4n+3}{n-1}$ converge?

And / But

$\sum_{n=0}^{\infty} \frac{4n+3}{n-1}$ diverge?

$\lim_{n\to\infty} \frac{4n+3}{n-1} = 4$ (Marilyn) Why?

$n=2$	$n=3$	$n=4$	$n=10$	$n=100$	$n=\infty$
11	7.5	$6.\overline{3}$	4.7	4.07	4

- These terms are approaching the # 4 so the lim $\boxed{\text{converges}}$

$\sum_{n=2}^{\infty} \frac{4n+3}{n-1} = 11 + 7.5 + 6.3 + \ldots + 4.7 + \ldots + 4.07 + \ldots + 4 + \ldots$

- The summation just keeps adding & adding so the Σ $\boxed{\text{diverges}}$

Infinite Series

Does $\sum_{n=0}^{\infty} e^{1/x}$ converge or diverge?

$\lim_{n\to\infty} e^{1/x} = e^{1/\infty} = e^0 = 1$

$\sum_{n=0}^{\infty} e^{1/x}$ diverges because $\lim_{n\to\infty} e^{1/x} = 1 \neq 0$.

Infinite Series

What does a Telescoping/Harmonic Series look like?

How do you determine if it converges or diverges?

Telescoping/Harmonic: Can be written $\sum_{n=1}^{\infty} a_n - a_{n+1}$

To find the summation you write out a bunch of terms & cancel what you can

converges: when you cancel everything & are left with a number.

diverge: After you cancel what you can you are still adding to ∞.

Infinite Series

Determine if the Harmonic Series Diverges or converges.

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$ Partial Fraction Decomp

$1 = A(n+1) + B(n)$ $\boxed{\text{let } n=-1}$ $1=-B$, $B=1$ $\boxed{\text{let } n=0}$ $1=A$

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

$\left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \ldots + \left(\cancel{\frac{1}{\infty}} - \cancel{\frac{1}{\infty+1}}\right)$ $\to 0$

$= \boxed{1}$ converges