

## Supplement: Sequences

## Infinite Series Day 1

Determine the limit of the sequence or state that the sequence diverges.

1.  $a_n = 4$

$$\lim_{n \rightarrow \infty} 4 = \boxed{4} \text{ converges}$$

▣ The limit of a constant is that constant.

2.  $a_n = 5 - \frac{9}{n^2}$

$$\lim_{n \rightarrow \infty} 5 - \frac{9}{n^2} = 5 - \frac{9}{\infty} = \boxed{5} \text{ converges}$$

3.  $c_n = -2^{-n}$

$$\lim_{n \rightarrow \infty} -2^{-n} = \lim_{n \rightarrow \infty} -\frac{1}{2^n} = -\frac{1}{2^\infty} = \boxed{0} \text{ converges}$$

4.  $z_n = \left(\frac{1}{3}\right)^n$

$$\lim_{n \rightarrow \infty} \left(\frac{1}{3}\right)^n = \lim_{n \rightarrow \infty} \frac{1^n}{3^n}$$

$$\lim_{n \rightarrow \infty} \frac{1}{3^n} = \frac{1}{3^\infty} = \boxed{0} \text{ converges}$$

5.  $a_n = \frac{(-1)^n n^2 + n}{4n^2 + 1}$

$$\begin{aligned} n = \text{even} \quad \lim_{n \rightarrow \infty} \frac{n^2 + n}{4n^2 + 1} &= \frac{1}{4} \\ n = \text{odd} \quad \lim_{n \rightarrow \infty} \frac{-n^2 + n}{4n^2 + 1} &= -\frac{1}{4} \end{aligned} \left. \vphantom{\lim} \right\} \text{different}$$

$\therefore$  diverges

6.  $a_n = \frac{n}{\sqrt{n^3 + 1}}$

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^3 + 1}} = \boxed{0} \text{ converges}$$

## Supplement: Sequences

## Infinite Series Day 1

Determine the limit of the sequence or state that the sequence diverges.

7.  $a_n = \cos(\pi n)$

$$n = \text{even} \quad \lim_{n \rightarrow \infty} \cos(\text{even}\pi) = 1$$

$$n = \text{odd} \quad \lim_{n \rightarrow \infty} \cos(\text{odd}\pi) = -1$$

different answers

 $\therefore a_n$  diverges

8.  $a_n = \frac{2n^2 - 4n + 5}{3n^2 + 2}$

$$\lim_{n \rightarrow \infty} \frac{2n^2 - 4n + 5}{3n^2 + 2} = \boxed{\frac{2}{3}}$$

converges

Determine if the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

9.  $a_n = \frac{1}{2n+1} \quad \lim_{n \rightarrow \infty} \frac{1}{2n+1} = 0$

$$n=1 \quad n=2 \quad n=3 \quad n=4 \quad n=5 \dots n=\infty$$
$$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \dots, 0$$

The function is decreasing  
& bounded from  $[0, \frac{1}{3}]$ 

10.  $a_n = \frac{3n^2}{n^2+2} \quad \lim_{n \rightarrow \infty} \frac{3n^2}{n^2+2} = 3$

$$n=1 \quad n=2 \quad n=3 \quad n=4 \dots n=\infty$$
$$\frac{3}{3}, \frac{12}{6}, \frac{27}{11}, \frac{48}{18}, \dots, 3$$
$$1, 2, 2\frac{6}{11}, 2\frac{2}{3}, \dots, 3$$

The function is increasing  
& bounded from  $[1, 3]$