BC Calculus

Sequences

Name________________________Pd._____

Infinite Series Day 1

Determine whether the sequence converges or diverges. If it converges find the limit.

1. $a_n = \dfrac{3+5n^2}{n+n^2}$ (Marilyn)

$$\lim_{n\to\infty} \frac{3+5n^2}{n+n^2} = \boxed{5}$$ ✓

∴ converges

2. $a_n = \dfrac{3^{n+2}}{5^n}$

$$\lim_{n\to\infty} \frac{3^n \cdot 3^2}{5^n} = \lim_{n\to\infty} \frac{9 \cdot 3^n}{5^n} = 9 \lim_{n\to\infty} \frac{3^n}{5^n}$$

$$9\left(\frac{3^\infty}{5^\infty}\right) = 9(0) = \boxed{0}$$ ✓

∴ converges

3. $a_n = \dfrac{(-1)^{n+1} n}{n+\sqrt{n}}$

$$\lim_{n\to\infty} \frac{-n}{n+\sqrt{n}} = -1$$ (if n=even)

✓ $\boxed{\therefore \text{diverges}}$

$$\lim_{n\to\infty} \frac{n}{n+\sqrt{n}} = 1$$ (if n=odd)

4. $a_n = \cos\left(\dfrac{2}{n}\right)$

$$\lim_{n\to\infty} \cos\left(\frac{2}{n}\right) = \cos\left(\frac{2}{\infty}\right) = \cos(0) = \boxed{1}$$ ✓

5. $a_n = \dfrac{\cos^2 n}{2^n}$

$$\lim_{n\to\infty} \frac{\cos^2 n}{2^n} = \frac{[-1,1]}{2^\infty} = \frac{\text{constant}}{\infty} = \boxed{0}$$ ✓

6. $a_n = \sqrt{\dfrac{n+1}{9n+1}}$ (Marilyn)

$$\lim_{n\to\infty} \sqrt{\frac{n+1}{9n+1}} = \frac{1}{\sqrt{9}} = \boxed{\frac{1}{3}}$$ ✓

Determine if the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

7. $a_n = \dfrac{1}{2n+3}$

n=1	n=2	n=3	n=4	n=5	n=6		n=∞
$\frac{1}{5}$,	$\frac{1}{7}$,	$\frac{1}{9}$,	$\frac{1}{11}$,	$\frac{1}{13}$,	$\frac{1}{15}$,	…	$\frac{1}{\infty}$

This function is decreasing and bounded $[0, \frac{1}{5}]$ ✓

8. $a_n = \dfrac{2n-3}{3n+4}$

$$\lim_{n\to\infty} \frac{2n-3}{3n+4} = \frac{2}{3}$$

n=1	n=2	n=3	n=4	n=5		n=∞
$\frac{-1}{7}$,	$\frac{1}{10}$,	$\frac{3}{13}$,	$\frac{5}{16}$,	$\frac{7}{19}$,	……	$\frac{2}{3}$

This function is increasing and bounded $[-\frac{1}{7}, \frac{2}{3}]$ ✓