BC Calculus Remember: $\lim_{n\to\infty} a_n =$ end behavior   Name Homework Guide Pd.____

Sequences   Infinite Series Day 1

Determine whether the sequence converges or diverges. If it converges find the limit.

1. $a_n = \frac{3+5n^2}{n+n^2}$ (Marilyn)

$$\lim_{n\to\infty} \frac{3+5n^2}{n+n^2} = 5$$

$\therefore$ converges

2. $a_n = \frac{3^{n+2}}{5^n}$

3. $a_n = \frac{(-1)^{n+1} n}{n+\sqrt{n}}$ alternating

$\lim_{n\to\infty}$ (If n=even) $\frac{-n}{n+\sqrt{n}} = -1$

$\lim_{n\to\infty}$ (If n=odd $\frac{n}{n+\sqrt{n}} = 1$

$\therefore$ diverges

4. $a_n = \cos\left(\frac{2}{n}\right)$

5. $a_n = \frac{\cos^2 n}{2^n}$

$$\lim_{n\to\infty} \frac{[\cos n]^2}{2^n} = \frac{[-1,1]}{2^\infty} = \frac{\text{constant}}{\infty} = 0$$

$\therefore$ Converges

6. $a_n = \sqrt{\frac{n+1}{9n+1}}$

Determine if the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

7. $a_n = \frac{1}{2n+3}$

$$\frac{d}{dn}\left[\frac{1}{2n+3}\right] = \frac{(2n+3)(0)-(1)(2)}{(2n+3)^2}$$

$$= \frac{-2}{(2n+3)^2} \quad \frac{\text{neg}}{\text{positive}} = \text{neg}$$

$\therefore$ function is decreasing

n=1 n=2 n=3... n=∞

$\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$, ... $\frac{1}{\infty}$

$[0, 1/5]$

8. $a_n = \frac{2n-3}{3n+4}$

Sequences Infinite Series Day 1

Review: End Behavior (Marilyn)

R1. $\lim\limits_{x\to-\infty}\frac{2x-1}{1+2x}$

A. -1
B. 0
C. 1
D. 2
E. nonexistant

$\lim\limits_{x\to-\infty}\frac{2x-1}{1+2x}=\frac{2}{2}=1$

R2. $\int_{\frac{\pi}{2}}^{x}\cos t\,dt$ $= \sin t\Big|_{\pi/2}^{x}$

A. $\cos x$
B. $-\sin x$
C. $\sin x-1$
D. $\sin x+1$
E. $-\sin x+1$

$=\sin x-\sin\frac{\pi}{2}$
$=\sin x-1$

R3. The radius of a sphere is increasing at a constant rate of 2 cm/sec. At the instant when the volume of the sphere is increasing at $32\pi\ cm^3/sec$, the surface area of the sphere is: $V_{sphere}=\frac{4}{3}\pi r^3$ $SA_{sphere}=4\pi r^2$

K: $\frac{dR}{dt}=2$
F: SA
W: $\frac{dV}{dt}=32\pi$

A. 8π
B. $\frac{32\pi}{3}$
C. 16π
D. 64π
E. $\frac{256\pi}{3}$

d: $\frac{dV}{dt}=\frac{4}{\cancel{3}}\pi\cdot\cancel{3}R^2\frac{dR}{dt}$
$32\pi=4\pi R^2(2)$
$32\pi=8\pi R^2$
$4=R^2$ $R=2$
$SA=4\pi(2)^2=4\pi(4)=16\pi$

R4. $\frac{d}{ds}\left[A=\frac{\sqrt{3}}{4}(5s-1)^2\right]$, what is the instantaneous rate of change of A with respect to s at s=1?

A. $2\sqrt{8}$
B. $2\sqrt{3}$
C. $\frac{5}{2}\sqrt{3}$
D. $4\sqrt{3}$
E. $10\sqrt{3}$

$\frac{dA}{ds}=\frac{\sqrt{3}}{4}\cdot2(5s-1)'\cdot(5)$
$\frac{dA}{ds}\Big|_{s=1}=\frac{\sqrt{3}}{\cancel{4}}\cdot10\cdot(5\cancel{-1})$
$10\sqrt{3}$

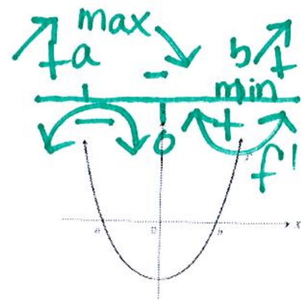
R5. What is the $\lim\limits_{x\to\ln2}g(x)$, if

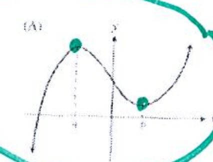
$g(x)=\begin{cases}e^x & if\ \ x>\ln2 \\ 4-e^x & if\ \ x\le\ln2\end{cases}$ Right / left

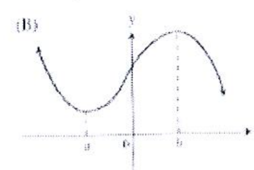
A. -2
B. $\ln2$
C. e^2
D. 2
E. nonexistant

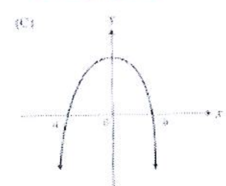
$\lim\limits_{x\to\ln2^+}=e^{\ln2}=2$
$\lim\limits_{x\to\ln2^-}=4-e^{\ln2}=2$

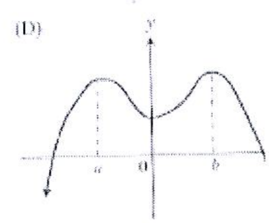
R6. The graph of f' is shown.
A possible graph of f is:

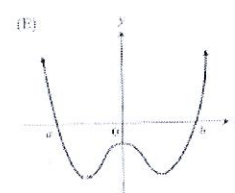


A.


B


C.


D


E.


Answers:

1. Converges to 5
2. Converges to 0
3. Diverges
4. Converges to 1
5. Converges to 0
6. Converges to 1/3
7. Decreasing and bounded [0, 1/5]
8. Increasing and bounded [-1/7, 2/3]

R1.C R2.C R3.C R4.E R5.D R6.A