Infinite series

Integral Test

→ When do you use it?

→ How do you use it?

Use for $\sum_{n=1}^{\infty} a_n$ $\boxed{}$ If a_n is positive, decreasing, & continuous

* Also if a_n is easy to integrate

Converges: $\lim_{R\to\infty} \int_1^R a_n =$ some number

diverges: $\lim_{R\to\infty} \int_1^R a_n = \infty$ OR $-\infty$

Infinite Series

Use the Integral test to determine if $\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln(n)}$ converges or diverges.

$\frac{1}{n\ln(n)}$ is positive, decreasing, & continuous $[2,\infty)$

$\lim_{R\to\infty} \int_2^R \frac{1}{n\ln(n)}\,dn$ $u = \ln(n)$, $du = \frac{1}{n}\,dn$

$\lim_{R\to\infty} \int_2^R \frac{1}{u}\,du$

$\lim_{R\to\infty} \ln|u|\Big]_2^R$

$\lim_{R\to\infty} \ln(\ln(n))\Big]_2^R$

→ $\lim_{R\to\infty} \ln(\ln R) - \ln(\ln 2)$

$\infty - \ln(\ln 2)$

∞

Diverges by the integral test b.c. $\int_1^{\infty} \frac{1}{n\ln(n)} = \infty$.

Infinite Series

P-Series Test

→ When do you use it?

→ How do you use it?

Use for $\sum_{n=1}^{\infty} \frac{1}{n^p}$

Converge: $p > 1$

diverge: $p \leq 1$

Infinite Series

Use p-series to determine if the following converge or diverge:

A. $\sum_{n=1}^{\infty} n^{-4/3}$

B. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

C. $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

A. $\sum_{n=1}^{\infty} n^{-4/3} = \sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ converges by p-series $p = \frac{4}{3} > 1$.

B. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges by p-series $p = \frac{3}{2} > 1$.

C. $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ diverges by p-series $p = \frac{1}{2} \leq 1$.

Infinite Series

Comparison Test

→ When do you use it?

→ How do you use it?

Use for $\sum_{n=1}^{\infty} a_n$

converge: If bigger series converges

diverge: If smaller series diverges

Must compare 2 series & prove bigger converges or smaller diverges

Infinite Series

Use Comparison test to show that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} \cdot 3^n}$ converges

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} \cdot 3^n}$

$\frac{1}{3^n} \boxed{>} \frac{1}{\sqrt{n} \cdot 3^n}$

$\sum_{n=1}^{\infty} \frac{1}{3^n} = \left(\frac{1}{3}\right)^n$ converges by geometric $R = 1/3$ so $|R| < 1$.

$\therefore \sum_{n=1}^{\infty} \frac{1}{\sqrt{n} \cdot 3^n}$ converges by comparison test.