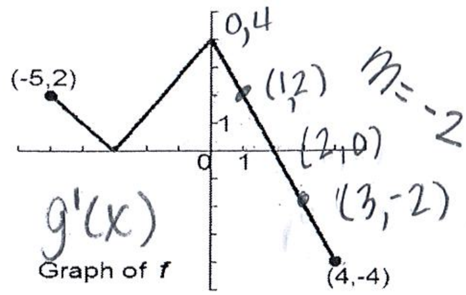
Non-Calculator
Graphs #3

Name____________________
Date_______________Pd.___

The function f is defined on the closed interval $[-5,4]$. The graph of f consists of three line Segments and is shown in the figure to the right.
Let $g(x)=\int_{-3}^{x} f(t)\,dt$.



$g'(x)=f(x)$ -5 + -3 + 0 + 2 − 5

$g''(x)=f'(x)$ − -3 + 0 −

a.) Find $g(3)$.

$$g(3)=\int_{-3}^{3} f(t)\,dt=\tfrac{1}{2}(5)(\cancel{4}^{\,2})-\tfrac{1}{2}(1)(2)$$
$$=10-1$$
$$g(3)=9$$ +1: answer

b.) On what open intervals contained in $-5<x<4$ is the graph of g both increasing and concave down? Give a reason for your answer.

$(-5,-3)\cup(0,2)$ Because on those intervals $g'(x)>0$ & $g''(x)<0$ +1: answer
+1: Reason

c.) The function h is defined by $h(x)=\frac{g(x)}{5x}$. Find $h'(3)$.

$g'(3)=f(3)=-2$

$$h'(x)=\frac{5x\cdot g'(x)-g(x)\cdot 5}{25x^2}$$ +2: h'(x)

$$h'(3)=\frac{15\cdot g'(3)-g(3)\cdot 5}{25(9)}=\frac{15(-2)-5(9)}{25(9)}=\frac{\cancel{5}(-6-9)}{\cancel{5}5(9)}=\frac{-15}{45}=-\frac{1}{3}$$ +1: answer

d.) The function p is defined by $p(x)=f(x^2-x)$. Find the slope of the line tangent to the graph of p at the point where $x=-1$.

$p'(x)=f'(x^2-x)\cdot[2x-1]$ +2: p'(x)

$p'(-1)=f'(1+1)\cdot[-2-1]$

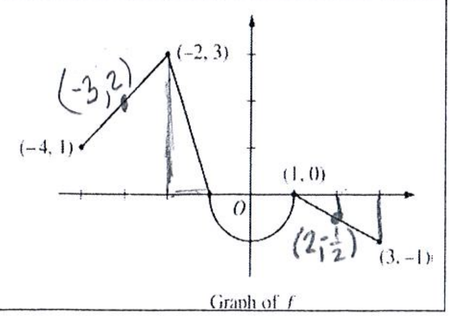
$p'(-1)=-3\cdot f'(2)=-3(-2)=6$ +1: answer

Non-Calculator
Graphs #4

Name________________
Date__________Period___

Let f be the continuous function defined on $[-4,3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given to the right. Let g be the function given by $g(x)=\int_1^x f(t)\,dt$.

$g'(x)=f(x)$
$g''(x)=f'(x)$



Graph of f

a.) Find the values of $g(2)$ and $g(-2)$.

$g(2)=\int_1^2 f(t)dt=\frac{1}{2}(1)(-\frac{1}{2})=-\frac{1}{4}$ +1: $g(2)$

$g(2)=g(-2)+\int_{-2}^{2} f(t)$

$-\frac{1}{4}=g(-2)+\frac{1}{2}(1)(3)-\frac{1}{2}\pi(1)^2-\frac{1}{2}(1)(\frac{1}{2})$ $\quad g(-2)=\frac{\pi}{2}-\frac{3}{2}$ +1: $g(-2)$

b.) For each of $g'(-3)$ and $g''(-3)$, find the value of state that it does not exist.

$g'(-3)=f(-3)=\underline{2}$ +1: $g'(-3)$ $\qquad g''(-3)=f'(-3)=\underline{1}$ +1: $g''(3)$

c.) Find the $x-$coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

g has a horizontal tangent where $\underline{g'(x)=0}$ +1: considers $g'(x)=0$

$-4 \; + \; -1 \; - \; 1 \; - \; 3$

x=−1 is a maximum because there g'(x) changes from positive to negative

+1: x=−1 & x=1

x=1 is neither because g'(x) does not change sign

+1: answers with justification

d.) For $-4<x<3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

g(x) has a point of inflection at x= −2, 0, & 1 because g'(x) changes signs at those points.

+1: Justification

$-4 \; + \; -2 \; - \; 0 \; + \; 1 \; - \; 3$

+1: answer