Name: ____________ Group Members: ____________

Exploration 6-2a: Another Form of the Fundamental Theorem

Date: ____________

Objective: Find the derivative of a definite integral from a fixed lower limit to a variable upper limit.

1. Let $f(x) = \int_1^x t^{1/2}\,dt$. Evaluate $f(9)$.

2. Sketch the graph of $y = t^{1/2}$. Show on the graph the geometrical meaning of $f(x)$.

3. Evaluate the integral to find an equation for $f(x)$ that does not involve the integral sign.

4. Differentiate both sides of the equation in Problem 3 to find an equation for $f'(x)$. Then tell how you could get $f'(x)$ quickly, in one step, simply by looking at the definition of $f(x)$ in Problem 1.

5. Let $g(x) = \int_3^x t^3\,dt$. Quick! Find $g'(x)$.

6. Let $h(x) = \int_2^{x^3} \cos t\,dt$. Evaluate the integral to find an equation for $h(x)$ that does not involve the integral sign.

7. Find $h'(x)$ using your answer in Problem 6. Observe the chain rule!

8. By observing the pattern in Problems 6 and 7, evaluate the following derivative *quickly*.

$$\frac{d}{dx}\left[\int_0^{\sin x} \tan^3(t^5)\,dt\right]$$

9. As a result of your work in Problems 1 through 8, you should be able to understand the **fundamental theorem of calculus in the derivative of an integral form,** specifically:

   If $g(x) = \int_a^x f(t)\,dt$, where a stands for a constant, then $g'(x) = f(x)$.

   Use this theorem to find $L'(x)$ quickly if

$$L(x) = \int_1^x \frac{1}{t}\,dt$$

10. Explain why $L(x)$ in Problem 9 cannot be evaluated directly by the fundamental theorem in the $g(b) - g(a)$ form using the power rule for antiderivatives.