Non-Calculator
Differential Equations #1

Name____________________
Date______________Pd_____

Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

a.) On the axis provided, sketch a slope field for the given differential equation at the six points indicated.

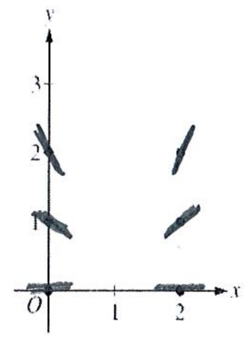
$(0,2) = \frac{4}{-1} = -4$ $\quad (2,2) = \frac{4}{1} = 4$

$(0,1) = \frac{1}{-1} = -1$ $\quad (2,1) = \frac{1}{1} = 1$

$(0,0) = \frac{0}{-1} = 0$ $\quad (2,0) = \frac{0}{1} = 0$

+1: zero slopes

+1: non-zero slopes



b.) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

Point: $f(2) = 3$

Slope: $\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{9}{2-1} = 9$

$y - 3 = 9(x-2)$ +1: tangent line equation

$y = 9(x-2) + 3$

$y(2.1) = 9(2.1-2) + 3$

$y(2.1) = 9(.1) + 3$

$y(2.1) = .9 + 3$

$y(2.1) = 3.9$ +1: approximation

c.) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(\overset{x}{2}) = \overset{y}{3}$.

$\frac{dy}{dx} = \frac{y^2}{x-1}$

$\frac{1}{y^2}dy = \frac{1}{x-1}dx$

+1: separation of variables

$\int y^{-2}dy = \int \frac{1}{x-1}dx$

$u = x-1$

$du = dx$

$\frac{y^{-1}}{-1} = \int \frac{1}{u}du$ +2: antiderivatives

$-\frac{1}{y} = \ln|x-1| + C$

$-\frac{1}{3} = \ln|2-1| + C$ +1: constant of integration & uses initial condition

$C = -\frac{1}{3}$

$-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$

$\frac{1}{y} = \frac{1}{3} - \ln|x-1|$

$y = \frac{1}{\frac{1}{3} - \ln|x-1|}$ +1: solves for y

$y = \frac{3}{1 - \ln|x-1|} = \frac{-3}{\ln|x-1| - 1}$

Non-Calculator
Differential Equations #2

Name____________________
Date______________ Pd.___

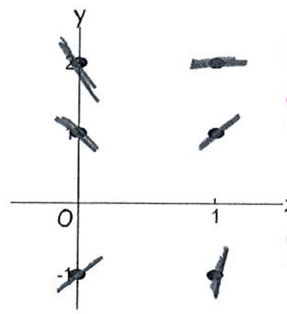
Consider the differential equation $\frac{dy}{dx} = 2x - y$

a.) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

$(0,2) = 0-2 = -2 \quad (1,2) = 2-2 = 0$

$(0,1) = 0-1 = -1 \quad (1,1) = 2-1 = 1$

$(0,-1) = 0+1 = 1 \quad (1,-1) = 2+1 = 3$



+1: slopes where $x=0$

+1: slopes where $x=1$

b.) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

$\frac{dy}{dx} = 2x - y$

$\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx}$

$\frac{d^2y}{dx^2} = 2 - (2x - y)$

$\frac{d^2y}{dx^2} = 2 - 2x + y$

+1: d^2y/dx^2

$\left.\frac{d^2y}{dx^2}\right|_{Q_2} = 2 - 2(\text{neg}) + \text{pos}$

$= 2 + \# + \# = \text{pos}$

$\frac{d^2y}{dx^2} > 0$ in Q_2 so concave up

+1: concave up with reason

c.) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

$\left.\frac{dy}{dx}\right|_{2,3} = 2(2) - 3 = 1$ +1: considers $\left.\frac{dy}{dx}\right|_{(2,3)}$

$f(2)$ is neither because $f'(2)$ does not equal 0.

+1: conclusion with justification

d.) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equations.

$y = mx + b$

$\frac{dy}{dx} = m$ +1: $\frac{dy}{dx} = m$

$2x - y = m$ +1: $2x - y = m$

$2x - (mx + b) = m$

$2x - mx - b = m$

$2x - mx - b - m = 0$

$(2-m)x - (m+b) = 0$

$2 - m = 0$

when $m = 2$

+1: answer

$-(2+b) = 0$

$-2 - b = 0$

$b = -2$