AP Calculus
Review Derivatives (1)
1-24: Calculate y':

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1. $y = (x^2 + x^3)^4$

$y' = 4(x^2 + x^3)^3 [2x + 3x^2]$

$y' = 4(2x + 3x^2)(x^2 + x^3)^3$

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2. $y = \dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt[5]{x^3}}$

$y = x^{-1/2} - x^{-3/5}$

$y' = -\dfrac{1}{2} x^{-3/2} + \dfrac{3}{5} x^{-8/5}$

$y' = \dfrac{-1}{2x^{3/2}} + \dfrac{3}{5x^{8/5}}$

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3. $y = \dfrac{x^2 - x - 2}{\sqrt{x}}$

$y = \dfrac{x^2}{x^{1/2}} - \dfrac{x}{x^{1/2}} - \dfrac{2}{x^{1/2}}$

$y = x^{3/2} - x^{1/2} - 2x^{-1/2}$

$y' = \dfrac{3}{2} x^{1/2} - \dfrac{1}{2} x^{-1/2} + x^{-3/2}$

$y' = \dfrac{3x^{1/2}}{2} - \dfrac{1}{2x^{1/2}} + \dfrac{1}{x^{3/2}}$

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4. $y = \dfrac{\tan x}{1 + \cos x}$

$y' = \dfrac{(1 + \cos x)\sec^2 x - \tan x(-\sin x)}{(1 + \cos x)^2}$

$y' = \dfrac{\sec^2 x(1 + \cos x) + \sin x \tan x}{(1 + \cos x)^2}$

⊛ Can keep going ☺ look at last page if interested

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5. $y = x^2 \sin(\pi x)$

$y' = x^2 \cos(\pi x)[\pi] + \sin(\pi x)[2x]$

$y' = \pi x^2 \cos(\pi x) + 2x \sin(\pi x)$

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6. $y = \dfrac{t^4 - 1}{t^4 + 1}$

$y' = \dfrac{(t^4 + 1)(4t^3) - (t^4 - 1)(4t^3)}{(t^4 + 1)^2}$

$y' = \dfrac{4t^7 + 4t^3 - 4t^7 + 4t^3}{(t^4 + 1)^2}$

$y' = \dfrac{8t^3}{(t^4 + 1)^2}$

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7. $y = \ln(x \ln x)$

$\dfrac{1}{x \ln x} \dfrac{d}{dx}[x \ln x]$

$\dfrac{1}{x \ln x}\left[x\left(\dfrac{1}{x}\right) + \ln x(1)\right]$

$\dfrac{1}{x \ln x}[1 + \ln x]$

$\dfrac{1 + \ln x}{x \ln x}$

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8. $y = \sqrt{x} \cos \sqrt{x} = x^{1/2} \cos x^{1/2}$

$y' = x^{1/2}(-\sin x^{1/2})\dfrac{d}{dx}[x^{1/2}] + \cos x^{1/2}\left[\dfrac{1}{2} x^{-1/2}\right]$

$y' = -\sqrt{x} \sin\sqrt{x}\left(\dfrac{1}{2} x^{-1/2}\right) + \cos\sqrt{x}\left(\dfrac{1}{2\sqrt{x}}\right)$

$y' = -\dfrac{\sqrt{x}\sin\sqrt{x}}{2\sqrt{x}} + \dfrac{\cos\sqrt{x}}{2\sqrt{x}}$

$= \dfrac{-\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}}{2\sqrt{x}}$

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9. $y = \dfrac{e^{\frac{1}{x}}}{x^2}$

$\dfrac{x^2 e^{\frac{1}{x}} \cdot \dfrac{d}{dx}[x^{-1}] - e^{\frac{1}{x}}[2x]}{x^4}$

$\dfrac{x^2 e^{1/x}(-x^{-2}) - 2xe^{\frac{1}{x}}}{x^4}$

$\dfrac{x^2 e^{1/x}\left(-\dfrac{1}{x^2}\right) - 2xe^{1/x}}{x^4}$

$\dfrac{-e^{1/x} - 2xe^{1/x}}{x^4} = \dfrac{-e^{1/x}(1 + 2x)}{x^4}$

10. $y = \ln \sec x$

$\dfrac{1}{\sec x} \dfrac{d}{dx}[\sec x]$

$\dfrac{1}{\sec x}[\sec x \tan x]$

$= \tan x$

11. $y = \cot(\csc x)$

$-\csc^2(\csc x)\dfrac{d}{dx}[\csc x]$

$-\csc^2(\csc x)[-\csc x \cot x]$

$\csc^2(\csc x)\csc x \cot x$

12. $y = e^{x\sec x}$

$e^{x\sec x}\dfrac{d}{dx}[x\sec x]$

$e^{x\sec x}[x\sec x\tan x + \sec x(1)]$

$e^{x\sec x}[x\sec x\tan x + \sec x]$

$e^{x\sec x}\cdot \sec x[x\tan x + 1]$

13. $y = 3^{x\ln x}$

$y' = 3^{x\ln x}\cdot \ln 3\cdot \dfrac{d}{dx}[x\ln x]$

$= 3^{x\ln x}\cdot \ln 3[x\cdot \tfrac{1}{x} + \ln x(1)]$

$= 3^{x\ln x}\cdot \ln 3[1 + \ln x]$

14. $y = \sec(1 + x^2)$

$\sec(1+x^2)\tan(1+x^2)[2x]$

$2x\sec(1+x^2)\tan(1+x^2)$

15. $y = \log_5(1 + 2x)$

$y' = \dfrac{1}{(1+2x)\ln 5}[2]$

$= \dfrac{2}{(1+2x)\ln 5}$

16. $y = (\cos x)^x$

$\ln y = x\cdot \ln \cos x$

$\dfrac{1}{y}[y'] = x\left(\dfrac{1}{\cos x}\right)(-\sin x) + \ln\cos x(1)$

$\dfrac{1}{y}[y'] = [-x\tan x + \ln\cos x]y$

$y' = [-x\tan x + \ln\cos x]\cos x^x$

17. $y = \dfrac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}$

$\ln y = 4\ln(x^2+1) - 3\ln(2x+1) - 5\ln(3x-1)$

$\dfrac{1}{y}[y'] = 4\left(\dfrac{1}{x^2+1}\right)(2x) - 3\left(\dfrac{1}{2x+1}\right)(2) - 5\left(\dfrac{1}{3x-1}\right)(3)$

$\dfrac{1}{y}[y'] = \dfrac{8x}{x^2+1} - \dfrac{6}{2x+1} - \dfrac{15}{3x-1}$

$y' = \left[\dfrac{8x}{x^2+1} - \dfrac{6}{2x+1} - \dfrac{15}{3x-1}\right]\left[\dfrac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}\right]$

18. $y = e^{\cos x} + \cos(e^x)$

$e^{\cos x}(-\sin x) + (-\sin e^x)e^x$

$-\sin x\, e^{\cos x} - \sin e^x\cdot e^x$

19. $y = 10^{\tan \pi\theta}$

$y' = 10^{\tan \pi\theta} \cdot \ln 10 \cdot \frac{d}{d\theta}[\tan \pi\theta]$

$= 10^{\tan \pi\theta} \cdot \ln 10 \cdot \sec^2 \pi\theta \cdot \frac{d}{d\theta}[\pi\theta]$

$= \pi \, 10^{\tan \pi\theta} \cdot \ln 10 \cdot \sec^2 \pi\theta$

20. $y = \cot(3x^2 + 5)$

$-\csc^2(3x^2+5)[6x]$

$-6x \csc^2(3x^2+5)$

21. $y = \sqrt{t \ln(t^4)} = \left(t \ln(t^4)\right)^{1/2}$

$\frac{1}{2}\left[t \ln(t^4)\right]^{-1/2}\left[t\left(\frac{1}{t^4}\right)(4t^3) + \ln(t^4)(1)\right]$

$\frac{1}{2}\left[t \ln(t^4)\right]^{-1/2}\left[4 + \ln(t^4)\right]$

$\dfrac{4 + \ln(t^4)}{2\sqrt{t \ln(t^4)}}$

22. $y = \tan^2(\sin\theta) = \left[\tan(\sin\theta)\right]^2$

$2\left[\tan(\sin\theta)\right]^1 \frac{d}{d\theta}\left[\tan(\sin\theta)\right]$

$2\tan(\sin\theta) \sec^2(\sin\theta) \frac{d}{d\theta}[\sin\theta]$

$2\tan(\sin\theta) \sec^2(\sin\theta) \cos\theta$

23. $y = (3x+2)^5(5-x)^8$

$(3x+2)^5(8)(5-x)^7(-1) + (5-x)^8(5)(3x+2)^4(3)$

$-8(3x+2)^5(5-x)^7 + 15(5-x)^8(3x+2)^4$

$(3x+2)^4(5-x)^7\left[-8(3x+2) + 15(5-x)\right]$

$(3x+2)^4(5-x)^7\left[-24x - 16 + 75 - 15x\right]$

$(3x+2)^4(5-x)^7\left[-39x + 59\right]$

24. $y = \left(\dfrac{2x-5}{x+4}\right)^8$

$8\left(\dfrac{2x-5}{x+4}\right)^7\left[\dfrac{(x+4)(2) - (2x-5)(1)}{(x+4)^2}\right]$

$\dfrac{8(2x-5)^7}{(x+4)^7} \cdot \dfrac{[2x+8 - 2x+5]}{(x+4)^2}$

$\dfrac{8(2x-5)^7(13)}{(x+4)^9} = \dfrac{104(2x-5)^7}{(x+4)^9}$

25-27: Find an equation of the tangent to the curve at the given point.

25. $y = 4\sin^2 x$, $\left(\dfrac{\pi}{6}, 1\right)$

$\left(\frac{\pi}{6}, 1\right)$ $y = 4[\sin x]^2$

$y' = 8\sin x \cos x$

$y'\left(\frac{\pi}{6}\right) = 8\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$y - 1 = 2\sqrt{3}\left(x - \frac{\pi}{6}\right)$

26. $y = \dfrac{x^2 - 1}{x^2 + 1}$, $(0, -1)$

$y' = \dfrac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$

$y' = \dfrac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2} = \dfrac{4x}{(x^2+1)^2}$

$y'(0) = \dfrac{4(0)}{(0^2+1)^2} = 0$

$y - -1 = 0(x - 0)$ $\boxed{y = -1}$

27. $y = \sqrt{1 + 4\sin x}$, $(0, 1)$

$y = (1 + 4\sin x)^{1/2}$

$y' = \frac{1}{2}(1 + 4\sin x)^{-1/2} \cdot 4\cos x$

$y' = \dfrac{4\cos x}{2\sqrt{1 + 4\sin x}}$

$y'(0) = \dfrac{4\cos(0)}{2\sqrt{1 + 4\sin(0)}} = \dfrac{4(1)}{2\sqrt{1}} = 2$

28-29: Suppose that h(x)=f(x)g(x) and F(x)=f(g(x)), where f(2)=3, g(2)=5, g'(2)=4, f'(2)= -2, and f'(5)=11.

28. Find $h'(2)$

$h(x) = f(x)g(x)$

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$h'(2) = f(2)g'(2) + g(2)f'(2)$

$= (3)(4) + (5)(-2)$

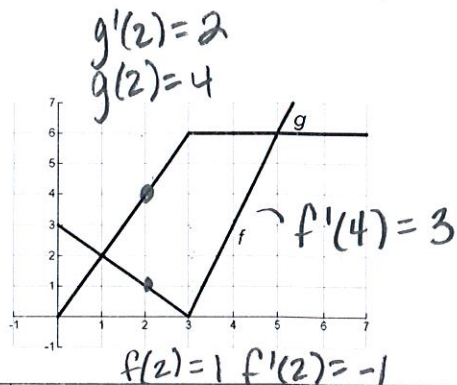$= 12 - 10$

$= 2$

29. Find $F'(2)$

$F(x) = f(g(x))$

$F'(x) = f'(g(x)) \cdot g'(x)$

$F'(2) = f'[g(2)] \cdot g'(2)$

$F'(2) = f'[5][4]$

$(11)(4)$

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30-32: Suppose f and g are functions whose graphs are shown, let P(x)=f(x)g(x), Q(x)=f(x)/g(x), and C(x)=f(g(x)).

$g'(2)=2$
$g(2)=4$

$f'(4)=3$

$f(2)=1 \quad f'(2)=-1$

$$Q(x)= \frac{f(x)}{g(x)}$$

30. Find $P'(2)$

$P(x) = f(x)g(x)$
$P'(x) = f(x)g'(x) + g(x)f'(x)$
$P'(2) = f(2)g'(2) + g(2)f'(2)$
$P'(2) = (1)(2) + (4)(-1)$
$P'(2) = 2-4$
$P'(2) = -2$

31. Find $Q'(2)$

$$Q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$Q'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$$

$$Q'(2) = \frac{4(-1) - (1)(2)}{4^2}$$

$$Q'(2) = \frac{-6}{16} = -\frac{3}{8}$$

32. Find $C'(2)$

$C(x) = f(g(x))$
$C'(x) = f'(g(x)) \cdot g'(x)$
$C'(2) = f'(g(2)) \cdot g'(2)$
$C'(2) = f'(4) \cdot [2]$
$C'(2) = [3][2]$
$C'(2) = 6$

33-40: Find f' in terms of g'

33. $f(x) = x^2 g(x)$

$x^2 g'(x) + g(x)[2x]$
$x^2 g'(x) + 2x g(x)$

34. $f(x) = g(x^2)$

$g'(x^2)[2x]$
$2x g'(x^2)$

35. $f(x) = [g(x)]^2$

$2[g(x)]^1 g'(x)$
$2g(x)g'(x)$

36. $f(x) = g(g(x))$

$g'[g(x)] \cdot g'(x)$

37. $f(x) = g(e^x)$

$g'[e^x] \cdot e^x$

38. $f(x) = e^{g(x)}$

$e^{g(x)} \cdot g'(x)$

39. $f(x) = \ln|g(x)|$

$\frac{1}{g(x)} \cdot g'(x)$

$\frac{g'(x)}{g(x)}$

40. $f(x) = g(\ln x)$

$g'[\ln x] \cdot \frac{1}{x}$

$\frac{g'[\ln x]}{x}$