

Approximating Derivatives or Avg. Rate of Change

CW Day 9

1. Use the given table to approximate the expressions below.

x	-2	-1	0	2	3	5	7
$f(x)$	-4	-1	4	10	11	15	19
$g(x)$	3	0	-1	-2	7	8	9
$h(x)$	18	22	28	39	50	44	36

$$\text{a.) } f'(-1) \frac{f(0) - f(-2)}{0 - (-2)}$$

$$\frac{4 - (-4)}{2} = \frac{8}{2} = \boxed{4}$$

$$\text{b.) } h'(5) \frac{h(7) - h(3)}{7 - 3}$$

$$\frac{36 - 50}{4} = \frac{-14}{4} = \boxed{-\frac{7}{2}}$$

$$\text{c.) } g'(1) \frac{g(2) - g(0)}{2 - 0}$$

$$\frac{-2 - 3}{2} = \boxed{-\frac{5}{2}}$$

$$\text{d.) } f'(7) \frac{f(7) - f(5)}{7 - 5}$$

$$\frac{19 - 15}{2} = \frac{4}{2} = \boxed{2}$$

$$\text{e.) } -5h'(4)$$

$$-5 \left[\frac{h(5) - h(3)}{5 - 3} \right] = -5 \left[\frac{44 - 50}{2} \right]$$

$$-5 \left[-\frac{6}{2} \right] = -5(-3) = \boxed{15}$$

$$\text{f.) } 3h'(0) - 2g'(7)$$

$$3 \left[\frac{h(2) - h(-1)}{2 - (-1)} \right] = 3 \left[\frac{39 - 22}{3} \right] = 17$$

$$2 \left[\frac{g(7) - g(5)}{7 - 5} \right] = 2 \left[\frac{9 - 8}{2} \right] = 1$$

$$17 - 1 = \boxed{16}$$

2. Explain why the expression in #1 may be undefined although numeric values were obtained by approximation. Provide as many reasons as you can.

There could be discontinuity, vertical tangent, a cusp, or corner at any of the above points.

3. Use the given graph to evaluate or approximate (whichever is appropriate) each expression.

Be sure to indicate approximate answers with the \approx symbol.

a.) $r'(1) \approx -2$

b.) $r'(2) \approx 0$

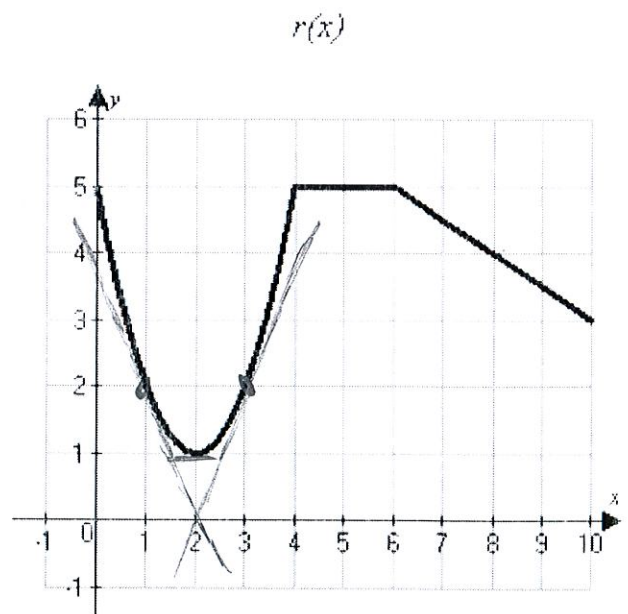
c.) $r'(3) \approx 2$

d.) $r'(4)$ dne

e.) $r'(5) \approx 0$

f.) $r'(6)$ dne

g.) $r'(8) \approx -\frac{1}{2}$



4. The number N of locations of a popular coffeehouse chain is given in the table. (The numbers of locations as of October 1 are given.)

Year	2004	2005	2006	2007	2008
N	8569	10,241	12,440	15,011	16,680

a.) Find the average rate of growth

From 2006 to 2008

From 2006 to 2007

From 2005 to 2006

Or $N'(2007)$

$$\frac{f(2008) - f(2006)}{2008 - 2006} = \frac{16,680 - 12,440}{2} = 4240/2 = \boxed{2120}$$

$$\frac{15,011 - 12,440}{1} = \boxed{2571}$$

$$\frac{12,440 - 10,241}{1} = \boxed{2199}$$

b.) Estimate the instantaneous rate of growth in 2006. What are its units.

$$\frac{f(2007) - f(2005)}{2007 - 2005} = \frac{15,011 - 10,241}{2} \frac{\text{locations}}{\text{year}} = \frac{4770}{2} = \boxed{2385 \frac{\text{locations}}{\text{year}}}$$

c.) Estimate the instantaneous rate of growth in 2007 and compare it with the growth rate in 2006. What do you conclude?

$$f'(2007) = \frac{2120 \text{ locations}}{\text{year}} \quad f'(2006) = \frac{2385 \text{ locations}}{\text{year}}$$

The number of chains being built per year is decreasing.

5. The cost of producing x ounces of gold from a new gold mine is $C=f(x)$ dollars.

a.) What is the meaning of the derivative $f'(x)$? What are its units?

$$f(x) = \text{Cost in dollars} \quad x = \text{ounces} \quad f'(x) = \frac{\text{Cost in dollars}}{\text{per ounce}}$$

b.) What does the statement $f'(800)=17$ mean?

At 800 ounces the cost is increasing 17 dollars.

c.) Do you think the values of $f'(x)$ will increase or decrease in the short term? What about the long term? Explain. In the short term it will decrease. Start up costs will spread over time. In the long run it will increase as the mine dries out.

6. The quantity (in pounds) of a gourmet ground coffee that is sold by a coffee company at a price of p dollars per pound is $Q=f(p)$.

a.) What is the meaning of the derivative $f'(8)$? What are its units?

$$f(p) = \text{Quantity} \quad p = \text{dollars} \quad f'(8) = \frac{\text{Quantity}}{\text{dollars}} = \text{cost for Quantity at 8 lbs.}$$

b.) Is $f'(8)$ positive or negative? Explain

At 8 lbs the cost should be decreasing.

7. Let $T(t)$ be the temperature (in $^{\circ}\text{F}$) in Phoenix t hours after midnight on September 10, 2008. The table shows values of the function recorded every two hours. What is the meaning of $T'(8)$? Estimate its value.

t	0	2	4	6	8	10	12	14
T	82	75	74	75	84	90	93	94

$$T'(8) = \frac{T(10) - T(6)}{10 - 6} = \frac{90 - 75}{4} = \frac{15^{\circ}\text{F}}{4 \text{ hour}} = 3.75^{\circ}\text{F per hour}$$

The temperature at 8:00 is increasing 3.75°F per hour.