AP Calculus
Review for Application of Integration Test

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Use the figure at the right to write an integral expression that represents the given. Do not evaluate.

1. The area of $\mathfrak{R}_{1}$
2. The area of $\mathfrak{R}_{2}$
3. The area of $\mathfrak{R}_{3}$

4. The area of $\mathfrak{R}_{4}$
5. The volume of $\Re_{2}$ revolved about the x-axis.
6. The volume of $\Re_{3}$ revolved about the $y$-axis.
7. $\quad$ The volume of $\Re_{1}$ revolved about the $x=a$.
8. The volume of $\Re_{2}$ revolved about the $y=-1$.
9. The volume of $\Re_{4}$ revolved about the x-axis.
10. The volume of $\Re_{5}$ revolved about the $y=1$.

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11. Compute the area of the region bounded by the functions $x=y+4$ and $x=y^{2}-2$.

## No Calculator


12. Compute the area of the region bounded by $y=x^{2}$ and $y=\sqrt{x}$

## No Calculator


13. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated lines. $\quad y=\sqrt{x}, y=0$, and $x=4$ Set up each. DO NOT INTEGRATE!!
A. The $x$-axis
B. The $y$-axis

C. The line $x=4$
D. The line $x=6$
E. The line $y=-3$
$\qquad$
14. Find the volume of the solid generated by the solid with base region bounded by the graphs of the equations about the indicated lines. $\quad y=\sqrt{x}, y=0$, and $x=4$ Set up each. DO NOT INTEGRATE!!
A. Cross sections perpendicular to x-axis are squares.
B. Cross sections perpendicular to the $x$-axis are semi-circles.

C. Cross sections perpendicular to the $x$-axis are rectangles with width being on the base and length is 3 times the width.
D. Find the region $\mathfrak{R}$ is the base of a solid, at each $x$ the cross section perpendicular to the $x$-axis has area $A(x)=\sin \left(\frac{\pi}{4} x\right)+1$. Find the volume of the solid.
E. Cross sections perpendicular to $y$-axis are equilateral triangles.
F. Cross sections perpendicular to y-axis are isosceles triangles with leg on the base.
15. The base of a solid is the region in the first quadrant enclosed by the parabola $y=3 x^{2}$, the line $x=1$ and the $x$-axis. Each cross section of the solid perpendicular to the $x$-axis is a square. The volume of the solid is:
A. $\pi$
B. $\frac{9}{5}$
C. $\frac{9 \pi}{5}$
D. $\frac{16}{5}$
E. 1
16. The base of a solid is the region enclosed by the parabola $y=x^{2}-2 x+4$ and $y=1+2 x$. If each cross section of the solid perpendicular to the $\mathbf{x}$-axis is a rectangle with a height that is 3 times the width, what is the volume of the solid?
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Calculators may be used on this portion of the test.
Answer the following.
17. Find the area bounded by the curves $-x+y=2$ and $y=-x^{2}+4 x+2$.

18. Find the area bounded by the curves $y=2 \sin x$ and $y=\tan x,-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

19. A region is bounded by the curve $y=x^{2}+2, y=6$, and $x=0$. Find the volume generated by revolving the region about the $y$-axis.

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20. Let $R$ be the region in the firs $\dagger$ quadrant bounded by the x-axis and the graphs of
$y=\ln x$ and $y=5-x$, as shown in the
figure to the right.

a.) Find the area of $R$.
b.) Region $R$ is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression involving one or more integrals that gives the volume of the solid.
c.) The horizontal line $y=k$ divides $R$ into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of $k$.

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Solve using separation of variables.
21. $y^{\prime}=x y^{2}$

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22. $y^{\prime}=0.5 x y$
23. $y^{\prime}=9 y$
24. $y^{\prime}=2(4-y)$
25. $\frac{d y}{d x}=2 \sqrt{y}$
26. $y^{\prime}=y^{2}\left(1-x^{2}\right)$

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Find the particular solution using the given point.
27. $y^{\prime}=2 y-4$ $(1,4)$
28. $y^{2} \frac{d y}{d x}=x^{-3} \quad(2,0)$

Find the average value of the function on the given interval. (Integrate by hand)
29. $f(x)=x^{2} ;[-1,1]$
30. $g(x)=\cos x,\left[0, \frac{\pi}{2}\right]$
31. $f(t)=t \sqrt{1+t^{2}},[0,5]$
32. $g(x)=x^{2} \sqrt{1+x^{3}},[0,2]$

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33. Consider the differential equation $\frac{d y}{d x}=(3-y) \cos x$. Let $y=f(x)$ be the particular solution to the differential equation with the initial condition $f(0)=1$. The function $f$ is defined for all real numbers.
a.) A portion of the slope field of the differential equation is given to the right. Sketch the solution curve through the point $(0,1)$

b.) Write an equation for the line tangent to the solution curve in part (a) at the point $(0,1)$. Use the equation to approximate $f(0.2)$.
c.) Find $y=f(x)$, the particular solution to the differential equation with the initial condition $f(0)=1$.

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## Answers:

1. $\int_{0}^{c} f(x)-g(x) d x$
2. $\int_{c}^{a} g(x)-f(x) d x$
3. $\int_{0}^{b} g^{-1}(y) d y$
4. $\int_{0}^{b} a-f^{-1}(y) d y$
5. $\pi \int_{c}^{a}[g(x)]^{2}-[f(x)]^{2} d x$
6. $\pi \int_{0}^{b}\left[g^{-1}(y)\right]^{2} d y$
7. $\pi \int_{-b}^{0}\left[a-f^{-1}(y)\right]^{2}-\left[a-g^{-1}(y)\right]^{2} d y$
8. $\pi \int_{c}^{a}[g(x)+1]^{2}-[f(x)+1]^{2} d x$
9. $\pi \int_{c}^{a}[f(x)]^{2} d x$
10. 

$\pi \int_{0}^{c}[1-f(x)]^{2}-1 d x$
11. $\frac{125}{6}$
12. $\frac{1}{3}$
13.
a) $\pi \int_{0}^{4} x d x$ b) $\pi \int_{0}^{2} 16-y^{4} d y$ c) $\pi \int_{0}^{2}\left(4-y^{2}\right)^{2} d y \quad$ d) $\pi \int_{0}^{2}\left(6-y^{2}\right)^{2}-4 d y \quad$ e) $\pi \int_{0}^{4}(\sqrt{x}+3)^{2}-9 d x$
14.
a) $\int_{0}^{4} x d x$
b) $\frac{\pi}{8} \int_{0}^{4} x d x \quad$ c) $3 \int_{0}^{4} x d x$
d) $\int_{0}^{4} A(x) d x$
e) $\frac{\sqrt{3}}{4} \int_{0}^{2}\left(4-y^{2}\right)^{2} d y$ f) $\frac{1}{2} \int_{0}^{2}\left(4-y^{2}\right)^{2} d y$
15. $B$
16. 3.2
17. 4.5
18. .614
19. 25.132
20.
a) 2.985
b) $\int_{1}^{3.6934}(\ln (x))^{2} d x+\int_{3.6934}^{5}(5-x)^{2} d x$
c) $1.493=\int_{0}^{k} 5-y-e^{y} d y$
21.

$$
y=\frac{-1}{\frac{1}{2} x^{2}+C}
$$

22. 

$$
y=C e^{\frac{1}{4} x^{2}}
$$

23. $y=C e^{9 x}$
24. 

$y=\frac{1}{C e^{2 x}}+4$
25. $y=(x+c)^{2}$
26. $y=\frac{-1}{x-\frac{1}{3} x^{3}+C}$
27. $y=2 e^{2 x-2}+2$
30. $\frac{2}{\pi}$
28. $y=\sqrt[3]{\frac{-3}{2 x^{2}}+\frac{3}{8}}$
29. $\frac{1}{3}$
31. $\frac{1}{5}\left((\sqrt{26})^{3}-1\right)$
32. $\frac{26}{9}$
33. a)

b) $\quad 1.4$

$$
y=3-2 e^{-\sin x}
$$

