AP Calculus
Review for Application of Integration Test

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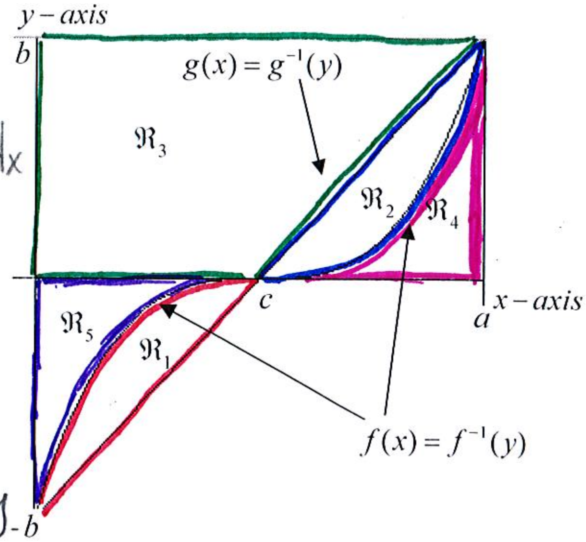
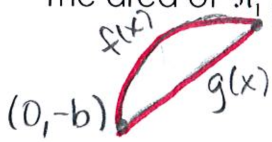
Use the figure at the right to write an integral expression that represents the given. **Do not evaluate.**



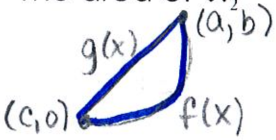

1. The area of $\Re_1$




$A=\int_{x_1}^{x_2} \text{top}-\text{bottom}\, dx$

$A=\int_0^c f(x)-g(x)\,dx$

2. The area of $\Re_2$




$A=\int_{x_1}^{x_2} \text{top}-\text{bottom}\, dx$

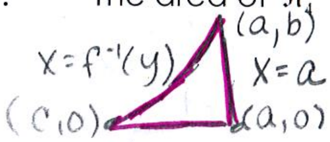
$A=\int_c^a g(x)-f(x)\,dx$

3. The area of $\Re_3$


$A=\int_{y_1}^{y_2} \text{right}-\text{left}\, dy$

$A=\int_0^b g^{-1}(y)-0\; dy$

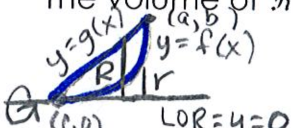
4. The area of $\Re_4$




$A=\int_{y_1}^{y_2} \text{right}-\text{left}\; dy=\int_0^b a-f^{-1}(y)\; dy$

5. The volume of $\Re_2$ revolved about the x-axis. dx

Hole

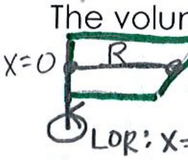



$R=g(x)-0$
$r=f(x)-0$

$V=\pi\int_{x_1}^{x_2} R^2-r^2\,dx=\pi\int_c^a (g(x)-0)^2-(f(x)-0)^2\,dx$

6. The volume of $\Re_3$ revolved about the y-axis. dy

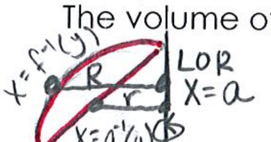
Solid




$X=g^{-1}(y)$ $R=g^{-1}(y)-0$ $V=\pi\int_{y_1}^{y_2} R^2\,dy=\pi\int_0^b (g^{-1}(y)-0)^2\,dy$

7. The volume of $\Re_1$ revolved about the x = a. dy

Hole

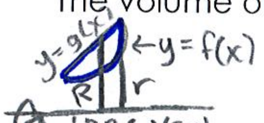



$R=a-f^{-1}(y)$
$r=a-g^{-1}(y)$

$V=\pi\int_{y_1}^{y_2} R^2-r^2\,dy=\pi\int_{-b}^0 (a-f^{-1}(y))^2-(a-g^{-1}(y))^2$

8. The volume of $\Re_2$ revolved about the y = −1. dx

Hole

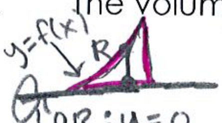



$R=g(x)--1$
$r=f(x)--1$

$V=\pi\int_{x_1}^{x_2} R^2-r^2\,dx=\pi\int_c^a (g(x)+1)^2-(f(x)+1)^2\,dx$

9. The volume of $\Re_4$ revolved about the x-axis. dx

Solid

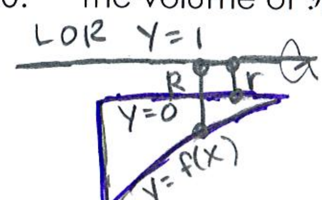



$R=f(x)-0$

$V=\pi\int_{x_1}^{x_2} R^2\,dx=\pi\int_c^a (f(x)-0)^2\,dx$

10. The volume of $\Re_5$ revolved about the y = 1. dx

hole




$R=1-f(x)$
$r=1-0$

$V=\pi\int_{x_1}^{x_2} R^2-r^2\,dx=\pi\int_0^c (1-f(x))^2-(1-0)^2\,dx$

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11. Compute the area of the region bounded by the functions $x = y+4$ and $x = y^2 - 2$.

| $x=y^2-2$ | $y$ |
|---|---|
| 7 | 3 |
| 2 | 2 |
| -1 | 1 |
| -2 | 0 |
| -1 | -1 |
| 2 | -2 |
| 7 | 3 |

| $x=y+4$ | $y$ |
|---|---|
| 7 | 3 |
| 6 | 2 |
| 5 | 1 |
| 4 | 0 |
| 3 | -1 |
| 2 | -2 |

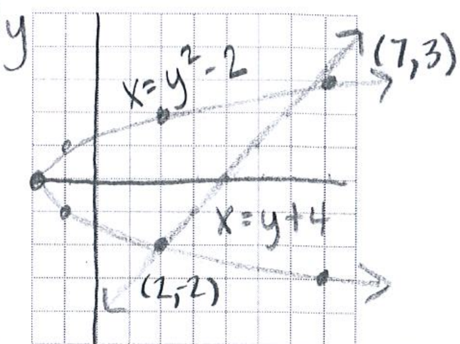
$A = \int_{y_1}^{y_2} \text{right} - \text{left}\, dy$

$= \int_{-2}^{3} y+4-y^2+2\, dy$

$= \int_{-2}^{3} -y^2+y+6$

$-\frac{y^3}{3} + \frac{y^2}{2} + 6y \Big|_{-2}^{3}$

$\left(-\frac{27}{3}+\frac{9}{2}+18\right)-\left(\frac{8}{3}+\frac{4}{2}-12\right) = -9+\frac{9}{2}+18-\frac{8}{3}-2+12 = 19+\frac{9}{2}-\frac{8}{3} = \frac{114}{6}+\frac{27}{6}-\frac{16}{6} = \boxed{\frac{125}{6}}$

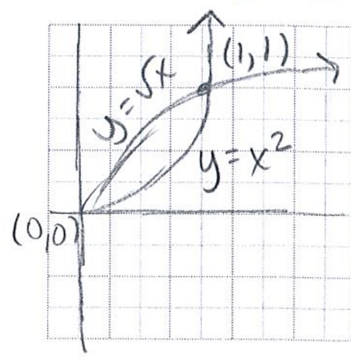



12. Compute the area of the region bounded by $y = x^2$ and $y = \sqrt{x}$

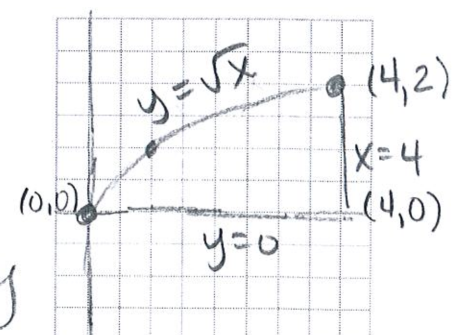
$A = \int_{x_1}^{x_2} \text{top} - \text{bottom}\, dx$

$A = \int_0 x^{1/2} - x^2\, dx$

$\frac{2}{3}x^{3/2} - \frac{x^3}{3} \Big|_0^1 = \frac{2}{3} - \frac{1}{3} - 0 + 0 = \boxed{\frac{1}{3}}$



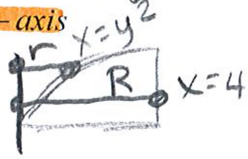

13. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated lines. $y = \sqrt{x}$, $y = 0$, and $x = 4$
Set up each. **DO NOT INTEGRATE!!**




dx A. The $x-axis$

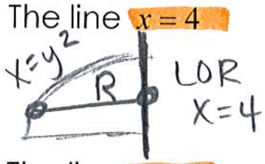

$\pi \int_0^4 (\sqrt{x}-0)^2 dx$

dy B. The $y-axis$



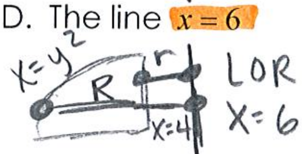

$\pi \int_0^2 (4-0)^2 - (y^2-0)^2 dy$

dy C. The line $x = 4$



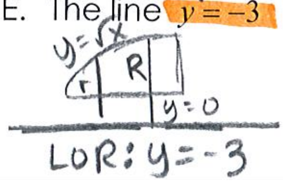

$\pi \int_0^2 (4-y^2)^2 dy$

dy D. The line $x = 6$




$\pi \int_0^2 (6-y^2)^2 - (6-4)^2 dy$

dx E. The line $y = -3$




$\pi \int_0^4 (\sqrt{x}+3)^2 - (0+3)^2 dx$

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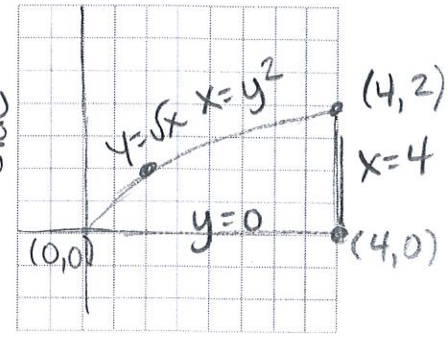
14. Find the volume of the solid generated by the solid with base region bounded by the graphs of the equations about the indicated lines. $y=\sqrt{x}$, $y=0$, and $x=4$

Set up each. **DO NOT INTEGRATE!!**

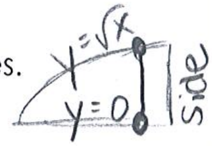
$\int \text{Area} = \text{Volume}$




dx A. Cross sections perpendicular to x-axis are squares.

$\int_{x_1}^{x_2} (\text{side})^2$   side = top − bottom

$\int_0^4 (\sqrt{x}-0)^2\,dx = \int_0^4 x\,dx$




dx B. Cross sections perpendicular to the x-axis are semi-circles.

$\frac{\pi}{8}\int_{x_1}^{x_2} \text{side}^2\,dx = \frac{\pi}{8}\int_0^4 (\sqrt{x}-0)^2\,dx$

$= \frac{\pi}{8}\int_0^4 x\,dx$

dx C. Cross sections perpendicular to the x-axis are rectangles with width being on the base and length is 3 times the width.

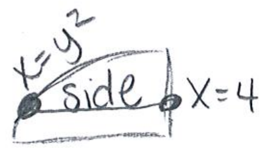

$\int_{x_1}^{x_2} w\cdot l = \int_0^4 (\sqrt{x}-0)\,3(\sqrt{x}-0)\,dx = 3\int_0^4 x\,dx$

dx D. Find the region $\mathfrak{R}$ is the base of a solid, at each $x$ the cross section perpendicular to the $x-axis$ has area $A(x)=\sin\left(\frac{\pi}{4}x\right)+1$. Find the volume of the solid.

$\int_0^4 A(x)\,dx$

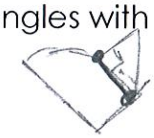
dy E. Cross sections perpendicular to y-axis are equilateral triangles.

$\frac{\sqrt{3}}{4}\int_{y_1}^{y_2} (\text{side})^2\,dy = \frac{\sqrt{3}}{4}\int_0^2 (4-y^2)^2\,dy$



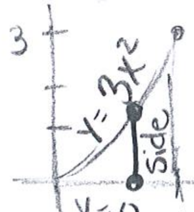

dy F. Cross sections perpendicular to y-axis are isosceles triangles with leg on the base.

$\frac{1}{2}\int_{y_1}^{y_2} (\text{side})^2\,dy = \frac{1}{2}\int_0^2 (4-y^2)^2\,dy$



dx 15. The base of a solid is the region in the first quadrant enclosed by the parabola $y=3x^2$, the line x = 1 and the x-axis. Each cross section of the solid perpendicular to the x-axis is a square. The volume of the solid is:

A. $\pi$  B. $\frac{9}{5}$  C. $\frac{9\pi}{5}$  D. $\frac{16}{5}$  E. 1

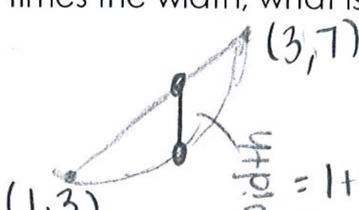



$\int_{x_1}^{x_2} \text{side}^2\,dx$

$\int_0^1 (3x^2-0)^2\,dx$

$=1.8=\frac{9}{5}$

dx 16. The base of a solid is the region enclosed by the parabola $y=x^2-2x+4$ and y = 1+ 2x. If each cross section of the solid perpendicular to the **x-axis** is a rectangle with a height that is 3 times the width, what is the volume of the solid?




$\int_{x_1}^{x} w\cdot h = \int_1^3 (-x^2+4x-3)\cdot 3(-x^2+4x-3)\,dx$

$= 3\int_1^3 (-x^2+4x-3)^2\,dx = 3.2$

$=1+2x-x^2+2x-4$

$=-x^2+4x-3$

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**Calculators** may be used on this portion of the test.

Answer the following.

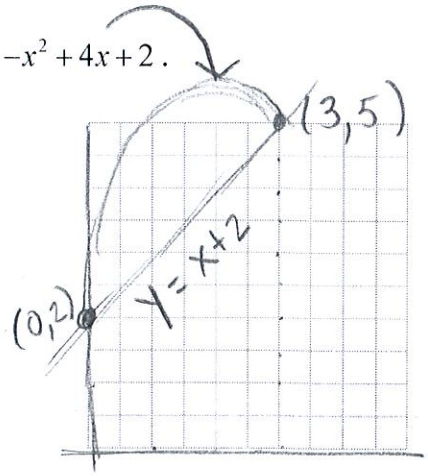
17. Find the area bounded by the curves -x + y = 2 and $y = -x^2 + 4x + 2$.

$A = \int_{x_1}^{x_2} \text{top} - \text{bottom}\, dx$

$A = \int_0^3 -x^2 + 4x + 2 - x - 2\, dx$

$A = \int_0^3 -x^2 + 3x\, dx$

$A = 4.5 = \boxed{\frac{9}{2}}$

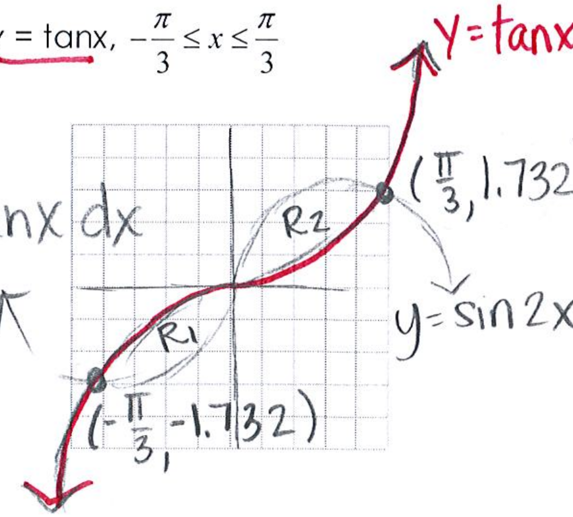



18. Find the area bounded by the curves y = 2sin x and y = tanx, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$

$A = R_1 + R_2$

$A = \int_{-\frac{\pi}{3}}^{0} \tan x - 2\sin x\, dx + \int_0^{\frac{\pi}{3}} 2\sin x - \tan x\, dx$

$= \boxed{.614}$




19. A region is bounded by the curve $y = x^2 + 2$, $y = 6$, and x=0. Find the volume generated by revolving the region about the y-axis.

dy

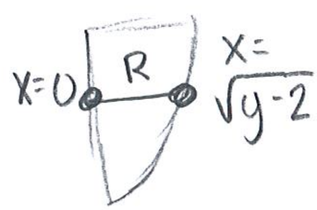
$\pi \int_{y_1}^{y_2} R^2\, dy$

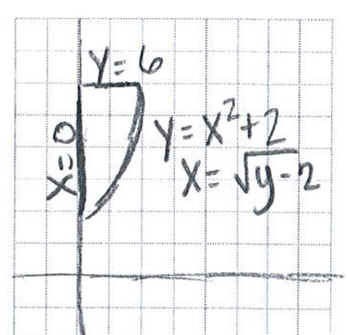
$\pi \int_2^6 (\sqrt{y-2} - 0)^2\, dy$

$\pi \int_2^6 y - 2\, dy$

$8\pi \approx 25.132$

$\approx 25.133$



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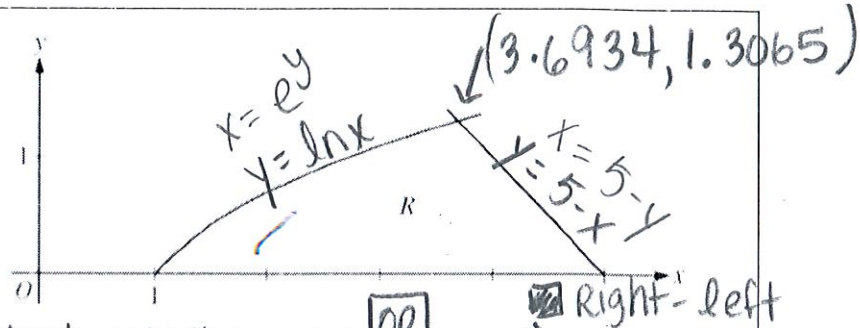
20. Let $R$ be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure to the right.




OR

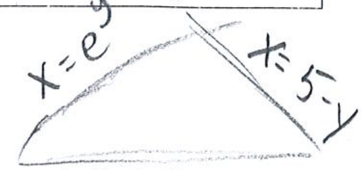
a.) Find the area of $R$. You can do top - bottom

$$A = R_1 + R_2$$

$$= \int_1^{3.6934} (\ln x - 0)\,dx + \int_{3.6934}^{5} (5 - x - 0)\,dx$$

$$2.1322 + .8536$$

$$2.985 \text{ OR } 2.986$$




$$\int_0^{1.3065} 5 - y - e^y\,dy$$

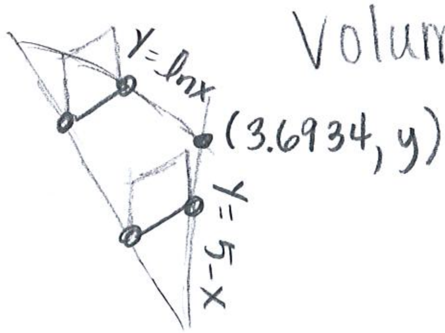
$$2.985 \text{ OR } 2.986$$

OR

b.) Region $R$ is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression involving one or more integrals that gives the volume of the solid.

dx

$$\text{Volume} = \int_1^{3.6934} (\ln x - 0)^2\,dx + \int_{3.6934}^{5} (5 - x - 0)^2\,dx$$




c.) The horizontal line $y = k$ divides $R$ into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$$A = 2.986$$

$$\tfrac{1}{2}A = 1.493$$

$$1.493 = \int_0^{k} (5 - y) - (e^y)\,dy$$

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Solve using separation of variables.

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21. $y' = xy^2$

$\frac{dy}{dx} = xy^2$

$\frac{1}{y^2} dy = x\,dx$

$\int y^{-2} dy = \int x\,dx$

$\frac{y^{-1}}{-1} = \frac{x^2}{2} + C$

$-\frac{1}{y} = \frac{1}{2}x^2 + C$

$-1 = y(\frac{1}{2}x^2 + C)$

$\boxed{y = \frac{-1}{\frac{1}{2}x^2 + C}}$

22. $y' = 0.5xy$

$\frac{dy}{dx} = .5xy$

$\frac{1}{y} dy = .5x\,dx$

$\int \frac{1}{y} dy = \int .5x\,dx$

$\ln|y| = \frac{1}{2}\frac{x^2}{2} + C$

$e^{\ln|y|} = e^{\frac{1}{4}x^2 + C}$

$|y| = Ce^{\frac{1}{4}x^2}$

$\boxed{y = Ce^{\frac{1}{4}x^2}}$

23. $y' = 9y$

$\frac{dy}{dx} = 9y$

$\frac{1}{y} dy = 9dx$

$\int \frac{1}{y} dy = \int 9dx$

$e^{\ln|y|} = e^{9x + C}$

$|y| = Ce^{9x}$

$\boxed{y = Ce^{9x}}$

24. $y' = 2(4 - y)$

$\frac{dy}{dx} = 2(4-y)$

$\frac{1}{4-y} dy = 2dx$

$u = 4-y$

$du = -dy$

$-du = dy$

$-\int \frac{1}{u} du = \int 2dx$

$-\ln|4-y| = 2x + C$

$e^{\ln|(4-y)^{-1}|} = e^{2x+C}$

$(4-y)^{-1} = Ce^{2x}$

$\frac{1}{4-y} = Ce^{2x}$

$4-y = \frac{1}{Ce^{2x}}$

$-y = \frac{1}{Ce^{2x}} - 4$

$\boxed{y = \frac{1}{Ce^{2x}} + 4}$

25. $\frac{dy}{dx} = 2\sqrt{y}$

$\frac{1}{\sqrt{y}} dy = 2dx$

$\int y^{-1/2} dy = \int 2dx$

$2y^{1/2} = 2x + C$

$y^{1/2} = \frac{2x + C}{2}$

$\sqrt{y} = x + C$

$\boxed{y = (x + C)^2}$

26. $y' = y^2(1 - x^2)$

$\frac{dy}{dx} = y^2(1-x^2)$

$\frac{1}{y^2} dy = 1 - x^2 dx$

$\int y^{-2} dy = \int 1 - x^2 dx$

$\frac{y^{-1}}{-1} = x - \frac{x^3}{3} + C$

$-\frac{1}{y} = x - \frac{x^3}{3} + C$

$\frac{y}{-1} = \frac{1}{x - \frac{x^3}{3} + C}$

$\boxed{y = \frac{-1}{x - \frac{x^3}{3} + C}}$

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Find the particular solution using the given point.

27. $y' = 2y - 4$ (1, 4)

$\frac{dy}{dx} = 2(y-2)$

$\frac{1}{y-2}\,dy = 2\,dx$

$u = y-2$
$du = dx$

$\int \frac{1}{u}\,du = \int 2\,dx$

$\ln|y-2| = 2x + C$

$\ln|4-2| = 2(1) + C$

$C = \ln 2 - 2$

$e^{\ln|y-2|} = e^{2x + \ln 2 - 2}$

$|y-2| = e^{2x-2} \cdot e^{\ln 2}$

$|y-2| = 2e^{2x-2}$

$y - 2 = \pm 2e^{2x-2}$

$y = \pm 2e^{2x-2} + 2$

✓ Initial condition

$\boxed{y = 2e^{2x-2} + 2}$

28. $y^2 \frac{dy}{dx} = x^{-3}$ (2, 0)

$\int y^2\,dy = \int x^{-3}\,dx$

$\frac{y^3}{3} = \frac{x^{-2}}{-2} + C$

$\frac{0^3}{3} = -\frac{1}{2(2)^2} + C$

$0 = \frac{-1}{8} + C$

$C = \frac{1}{8}$

$\frac{y^3}{3} = -\frac{1}{2x^2} + \frac{1}{8}$

$y^3 = \frac{-3}{2x^2} + \frac{3}{8}$

$\boxed{y = \sqrt[3]{-\frac{3}{2x^2} + \frac{3}{8}}}$

Find the average value of the function on the given interval. (Integrate by hand)

29. $f(x) = x^2$; [-1, 1]

$\frac{1}{1-{-1}} \int_{-1}^{1} x^2\,dx$

$\frac{1}{2}\left[\frac{x^3}{3}\Big|_{-1}^{1}\right] = \frac{1}{2}\left[\frac{1}{3} - \left(\frac{-1}{3}\right)\right]$

$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{3}\right] = \boxed{\frac{1}{3}}$

30. $g(x) = \cos x$, $[0, \frac{\pi}{2}]$

$\frac{1}{\frac{\pi}{2} - 0} \int_0^{\frac{\pi}{2}} \cos x\,dx$

$\frac{2}{\pi}\left[\sin x \Big|_0^{\frac{\pi}{2}}\right]$

$\frac{2}{\pi}\left[\sin\frac{\pi}{2} - \sin(0)\right] = \frac{2}{\pi}[1 - 0] = \boxed{\frac{2}{\pi}}$

31. $f(t) = t\sqrt{1+t^2}$, [0, 5]

$\frac{1}{5-0}\int_0^5 t\sqrt{1+t^2}\,dt$

$u = 1 + t^2$
$du = 2t\,dt$
$\frac{1}{2}du = t\,dt$
$u(0) = 1$
$u(5) = 26$

$\frac{1}{5} \cdot \frac{1}{2} \int u^{1/2}\,du$

$\frac{1}{5\cancel{10}}\left[\frac{\cancel{2}}{3}u^{3/2}\Big|_1^{26}\right]$

$\frac{1}{15}\left[(\sqrt{26})^3 - 1\right]$

32. $g(x) = x^2\sqrt{1+x^3}$, [0, 2]

$\frac{1}{2-0}\int_0^2 x^2\sqrt{1+x^3}\,dx$

$u = 1 + x^3$
$du = 3x^2\,dx$
$\frac{1}{3}du = x^2\,dx$
$u(0) = 1$
$u(2) = 9$

$\frac{1}{2} \cdot \frac{1}{3} \int_1^9 u^{1/2}\,du$

$\frac{1}{3\cancel{6}}\left[\frac{\cancel{2}}{3}u^{3/2}\Big|_1^9\right]$

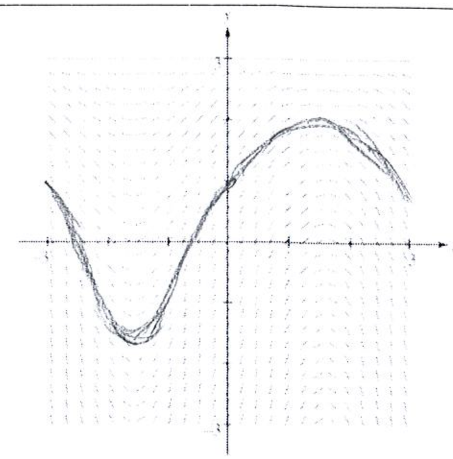
$\frac{1}{9}\left[(\sqrt{9})^3 - (\sqrt{1})^3\right] = \frac{1}{9}(27 - 1) = \boxed{\frac{26}{9}}$

AP Calculus
Review for Application of Integration Test

Name____________________
Date__________ Pd______ Day 9

33. Consider the differential equation $\frac{dy}{dx} = (3-y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0)=1$. The function $f$ is defined for all real numbers.

a.) A portion of the slope field of the differential equation is given to the right. Sketch the solution curve through the point $(0,1)$



b.) Write an equation for the line tangent to the solution curve in part (a) at the point $(0,1)$. Use the equation to approximate $f(0.2)$.

Point $(0,1)$

Slope: $\left.\frac{dy}{dx}\right|_{(0,1)} = (3-1)\cos(0) = 2(1) = 2$

$y - 1 = 2(x-0)$

$y = 2(x-0)+1$

$y(.2) = 2(.2)+1 = 1.4$

c.) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0)=1$.

$\frac{dy}{dx} = (3-y)\cos x$

$\frac{1}{3-y}\,dy = \cos x\,dx$

$\int \frac{1}{3-y}\,dy = \int \cos x\,dx$

$u = 3-y$

$du = -dy$

$-du = dy$

$-\int \frac{1}{u}\,du = \sin x + C$

$-\ln|3-y| = \sin x + C$

$-\ln|3-1| = \sin(0) + C$

$C = -\ln 2$

$-\ln|3-y| = \sin x - \ln 2$

$\ln|3-y| = \ln 2 - \sin x$

$e \qquad e$

$|3-y| = e^{\ln 2} e^{-\sin x}$

$|3-y| = 2e^{-\sin x}$

$3 - y = \pm 2e^{-\sin x}$

$-y = \pm 2e^{-\sin x} - 3$

$y = 3 \pm 2e^{-\sin x}$

Check initial condition

$\boxed{y = 3 - 2e^{-\sin x}}$