AP Calculus — Name________________________ Pd.___

Review #2 Curve Sketching — Curve Sketching Day 8

For problems 1-3, justify whether the Mean Value Theorem applies over the indicated interval. If it does apply, find the value(s) of c guaranteed in the interval (a,b).

1. $f(x) = x^{\frac{2}{3}}$ $[-1,1]$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$ $x=0$ is not differentiable

So $f(x)$ is not differentiable on $(-1,1)$

So MVT does not apply

2. $f(x) = \frac{x+1}{x}$ $\left[\frac{1}{2}, 2\right]$ $f'(x) = \frac{x(1)-(x+1)(1)}{x^2} = \frac{x-x-1}{x^2} = \frac{-1}{x^2} = \frac{\frac{3}{2}-3}{2-\frac{1}{2}}$

$f(x)$ is continuous $[\frac{1}{2}, 2]$

$f(x)$ is differentiable on $(\frac{1}{2}, 2)$

$\therefore$ MVT applies

$f(2) = \frac{3}{2}$

$f(\frac{1}{2}) = \frac{\frac{1}{2}+1}{\frac{1}{2}} = \frac{3}{2}\cdot\frac{2}{1} = 3$

$\frac{-1}{x^2} = \frac{-\frac{3}{2}}{\frac{3}{2}}$

$\frac{-1}{x^2} = -1$

$-x^2 = -1$ $x^2 = 1$

$x = \pm 1$

$\boxed{x = 1}$

3. $f(x) = \sin x$ $[0.\pi]$

$f'(x) = \cos x$

$f(x)$ is continuous $[0, \pi]$

$f(x)$ is differentiable $(0, \pi)$

$\therefore$ MVT applies

$\cos x = \frac{\sin \pi - \sin(0)}{\pi - 0}$

$\cos x = \frac{0}{\pi}$

$\cos x = 0$

$\boxed{x = \frac{\pi}{2}}$

Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

4. $f(x) = x^3 - x^2 - 6x + 2,$ $[0,3]$

$f'(x) = 3x^2 - 2x - 6$

1 $f(x)$ is continuous $[0,3]$

2 $f(x)$ is differentiable $(0,3)$

$\therefore$ Rolle's thm applies

Then $f'(x) = 0$ on $[0,3]$

3 $f(3) = 27 - 9 - 18 + 2 = 2$

$f(0) = 2$

$f(0) = f(3)$

$3x^2 - 2x - 6 = 0$

$(3x \quad)(x \quad)$

$y_1 = 3x^2 - 2x - 6$ $x = -1.12$ $\boxed{x = 1.786}$

5. $f(x) = \cos(2x),$ $\left[\frac{\pi}{8}, \frac{7\pi}{8}\right]$

$f'(x) = -2\sin(2x)$

1 $f(x)$ is continuous $[\frac{\pi}{8}, \frac{7\pi}{8}]$

2 $f(x)$ is differentiable $(\frac{\pi}{8}, \frac{7\pi}{8})$

3 $f(\frac{\pi}{8}) = \cos(2\cdot\frac{\pi}{8}) = \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ $f(7\pi/8) = \cos(2\cdot\frac{7\pi}{8}) = \cos(\frac{7\pi}{4}) = \frac{\sqrt{2}}{2}$

Then $f'(x) = 0$

$-2\sin(2x) = 0$

$\boxed{x = \frac{\pi}{2}}$

$\sin(☺) = 0$

☺ $= 0, \pi, 2\pi, 3\pi$

$2x = \frac{0}{2}, \frac{\pi}{2}, \frac{2\pi}{2}, \frac{3\pi}{2}$

$x = 0, \frac{\pi}{2}, \pi, 3\pi/2$

6. Consider the function f, whose formula and derivatives are given by

$$f(x)=\frac{x^2-4}{x+1},\quad f'(x)=\frac{x^2+2x+4}{(x+1)^2},\quad f''(x)=\frac{-6}{(x+1)^3}$$

a. Find and describe all of the vertical and horizontal asymptotes of this function, if any. Justify.

$f(x)=\frac{x^2-4}{x+1}$

$\boxed{\text{VA: } x=-1}$

$\boxed{\text{HA: none}}$

b. Find all of the roots of this function, if any.

$f(x)=\frac{x^2-4}{x+1}$

x int: $x^2-4=0$

$x^2=4$

$x=\pm 2$

$\boxed{(2,0)\quad(-2,0)}$

y int: $\frac{0^2-4}{0+1}=-4$

$\boxed{(0,-4)}$

c. Find and classify all of the local extrema of this function, if any. Show justification.

$f'(x)=\frac{x^2+2x+4}{(x+1)^2}$ → no crit#, → $x=-1$

+ | -1 VA | + $f'$

$\boxed{\text{no extrema}}$

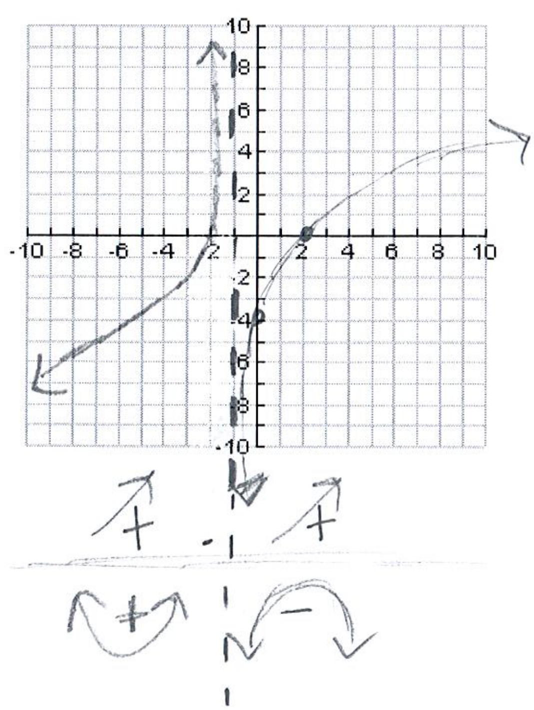
d. Find all of the inflection points of this function, if any. Show justification.

$f''(x)=\frac{-6}{(x+1)^3}$ → no possible POI, → $x=-1$

$f''(-2)=\frac{-}{-}$ + | -1 VA | − $f''(0)=\frac{-}{+}$ $f''$

$\boxed{\text{no POI}}$

e. Sketch the function and include all the features above.



| | |
|---|---|
| 7. $f(x) = 2x^3 - 5x^2 + 4x + 10$ | g. Sign Chart |
| a. f'(x)= $6x^2 - 10x + 4$ | max $\frac{2}{3}$ 1 min; $-\infty$ + − + $\infty$; + $5/6$ − |
| b. critical points | |
| c. Intervals increasing: $(-\infty, \frac{2}{3}) \cup (1, \infty)$<br>Intervals decreasing: $(\frac{2}{3}, 1)$ | h. At what x-value(s) does the graph have Any minimums or maximums? Justify your answer with the first and second derivatives tests. |
| d. f''(x)= $12x - 10$ | |
| e. Points of Inflection $x = 5/6$ | |
| f. Intervals of concave up $(5/6, \infty)$<br>Intervals of concave down $(-\infty, 5/6)$ | |

$\boxed{1st}$ $x = \frac{2}{3}$ is a maximum b.c. it is a critical # where f'(x) changes from pos to neg.

$x = 1$ is a minimum b.c it is a critical # where f' changes from neg to pos

$\boxed{2nd}$ $x = \frac{2}{3}$ is a max b.c it is a crit # & $f''(-\frac{1}{3}) = \text{neg}$

$x = \underline{1}$ is a min b.c it is a crit # & $f''(2) = \text{pos}$

$0 = 6x^2 - 10x + 4$ (÷2)

$0 = 2(3x^2 - 5x + 2)$

$0 = 2(3x - 2)(x - 1)$

$x = \frac{2}{3} \quad x = 1$

$f'(0) = (+)(-)(-) = +$

$f'(9/10) = (+)(+)(-) = -$

$f'(2) = (+)(+)(+) = +$

$0 = 12x - 10$

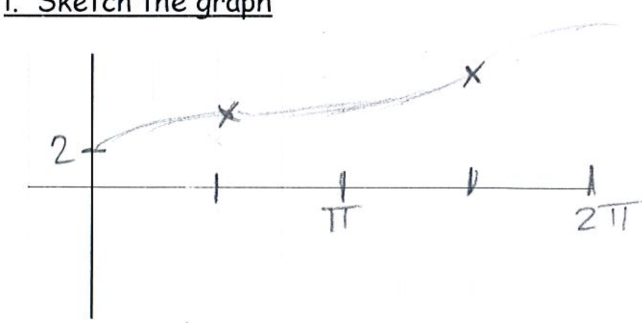
$12x = 10$

$x = \frac{5}{6}$

$f''(1) = +$

$f''(0) = -$

| | |
|---|---|
| 8. $f(x = \cos x + x, \quad [0, 2\pi]$ | g. Sign Chart |
| a. f'(x)= $-\sin x + 1$ | 0 + $\pi/2$ + $2\pi$; − $\pi/2$ + $3\pi/2$ − |
| b. critical points $x = \pi/2$ | |
| c. Intervals increasing: $(0, 2\pi)$<br>Intervals decreasing: none | h. At what x-value(s) does the graph have Any minimums or maximums? Justify your answer with the first and second derivatives tests. |
| d. f''(x) = $-\cos x$ | $x = \frac{\pi}{2}$ is a critical number & it is not extrema b.c. there is no sign change |
| e. Points of Inflection $\pi/2, 3\pi/2$ | |
| f. Intervals of concave up $(\pi/2, 3\pi/2)$<br>Intervals of concave down $(0, \pi/2)(\frac{3\pi}{2}, 2\pi)$ | i. Sketch the graph |



Find each limit.

1. $\lim_{x\to 1}\frac{\ln x}{x-1}$ $\frac{0}{0}$

$\lim_{x\to 1}\frac{1/x}{1}$

$\lim_{x\to 1}\frac{1}{x}=\frac{1}{1}=\boxed{1}$

2. $\lim_{x\to\infty}\frac{x^2+x}{\ln x}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{2x+1}{\cancel{\frac{1}{x}\cdot\frac{x}{1}}}\cdot\frac{x}{1}$

$\lim_{x\to\infty}2x^2+x=\infty+\infty=\boxed{\infty}$

3. $\lim_{x\to -1}\frac{x^2-x}{x+1}$ $\frac{2}{1}$ → not L'Hopitals Rule

VA at $x=-1$

$\lim_{x\to -1^+}\frac{(-.999)^2-(-.999)}{-.999+1}=\frac{+}{+}=+\infty$

$\lim_{x\to -1^-}\frac{(-1.001)^2-(-1.001)}{-1.001+1}=\frac{+}{-}=-\infty$

$\lim_{x\to -1}\frac{x^2-x}{x+1}=\boxed{dne}$

4. $\lim_{x\to 0^+}\sqrt{x}\ln x$ $0(-\infty)$

$\lim_{x\to 0^+}\frac{\ln x}{x^{-1/2}}$ $\frac{-\infty}{0}$

$\lim_{x\to 0^+}\frac{1/x}{-\frac{1}{2}x^{-3/2}}$

$\lim_{x\to 0^+}\frac{\frac{1}{x}}{\frac{-1}{2x^{3/2}}}\cdot\frac{2x^{3/2}}{-1}$

$\lim_{x\to 0^+}\frac{2x^{1/2}}{-1}$

$\frac{2\sqrt{0}}{-1}$

$=\boxed{0}$

5. $\lim_{x\to\infty}\frac{e^x}{x^3}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{e^x}{3x^2}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{e^x}{6x}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{e^x}{6}=\frac{\infty}{6}=\boxed{\infty}$

6. $\lim_{x\to 0^+}\left(1-x^2\right)^{1/x^2}$ $1^0$

$\ln y=\lim_{x\to 0^+}\ln(1-x^2)^{1/x^2}$

$\ln y=\lim_{x\to 0^+}\frac{1}{x^2}\ln(1-x^2)$

$\ln y=\lim_{x\to 0^+}\frac{\ln(1-x^2)}{x^2}$ $\frac{0}{0}$

$\ln y=\lim_{x\to 0^+}\frac{\frac{1}{1-x^2}\cdot(-\cancel{2x})}{\frac{\cancel{2x}}{1}}$ $\frac{1}{\cancel{2x}}$ $\frac{1}{\cancel{2x}}$

$\ln y=\lim_{x\to 0^+}\frac{-1}{1-x^2}$ $\ln y=\frac{-1}{1-0}$

$e^{\ln y}=e^{-1}$

$\boxed{y=e^{-1}}$