Calculus : Derivatives

Name ______________________________

Date ________________ Period ______

All problems on this review are to be worked without a calculator.

For problems 1-6, find $\frac{dy}{dx}$

1. $x^3 + y^3 = 8xy$

$3x^2 + 3y^2\frac{dy}{dx} = 8x\frac{dy}{dx} + y(8)$

$3y^2\frac{dy}{dx} - 8x\frac{dy}{dx} = 8y - 3x^2$

$\frac{dy}{dx} = \frac{8y - 3x^2}{3y^2 - 8x}$

2. $x^3 - xy + y^2 = 4$

$3x^2 - x\frac{dy}{dx} + y(-1) + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2y - x) = y - 3x^2$

$\frac{dy}{dx} = \frac{y - 3x^2}{2y - x}$

3. $x^2y + y^2x = -2$

$x^2\frac{dy}{dx} + y(2x) + y^2(1) + x(2y)\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$

$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$

4. $2\sin x \cos y = 1$

$2\sin x(-\sin y)\frac{dy}{dx} + \cos y \, 2\cos x = 0$

$-2\sin x \sin y\frac{dy}{dx} = -2\cos x \cos y$

$\frac{dy}{dx} = \frac{-2\cos x \cos y}{-2\sin x \sin y} = +\cot x \cot y$

5. $\sin x + 2\cos(2y) = 1$

$\cos x + 2(-\sin(2y))(2)\frac{dy}{dx} = 0$

$\cos x = 4\sin(2y)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{\cos x}{4\sin(2y)}$

6. $y = \sin(xy)$

$\frac{dy}{dx} = \cos(xy)\left[x\frac{dy}{dx} + y(1)\right]$

$\frac{dy}{dx} = x\cos(xy)\frac{dy}{dx} + y\cos(xy)$

$\frac{dy}{dx} - \frac{dy}{dx}x\cos(xy) = y\cos(xy)$

$\frac{dy}{dx}(1 - x\cos(xy)) = y\cos(xy)$

$\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$

7. Find $\frac{d^2y}{dx^2}$ for $x^2 - y^2 = 16$

$2x - 2y\frac{dy}{dx} = 0 \qquad \frac{dy}{dx} = +\frac{x}{y}$

$\frac{d^2y}{dx^2} = \frac{y(+1) - (+x)\frac{dy}{dx}}{y^2}$

$= \frac{\frac{y}{y}y - x\left(+\frac{x}{y}\right)}{y^2} = \frac{\frac{y^2 - x^2}{y}}{\frac{y^2}{1}} \cdot \frac{\frac{1}{y^2}}{\frac{1}{y^2}} = \frac{-1[x^2 - y^2]}{y^3} = \frac{-1(16)}{y^3} = \frac{-16}{y^3}$

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Name ______________________________

Date ________________ Period ______

8. Find an equation of the tangent line to the curve $x^3 + y^3 = 2xy$ at the point $(1,1)$. Point $(1,1)$

$3x^2 + 3y^2\frac{dy}{dx} = 2x\frac{dy}{dx} + y(2)$

$\boxed{y-1=-1(x-1)}$

Slope: $\left.\frac{dy}{dx}\right|_{(1,1)} = \underline{-1}$

$3y^2\frac{dy}{dx} - 2x\frac{dy}{dx} = 2y - 3x^2$

$\frac{dy}{dy} = \frac{2y-3x^2}{3y^2-2x}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{2-3}{3-2} = \frac{-1}{1} = -1$

For problems 9-10, find $\frac{d}{dx}\left(f^{-1}(x)\right)$ ***at*** $x = a$

9. $f(x) = \frac{1}{4}x^3 + x - 1$ *where* $a = 3$

$\frac{1}{f'[f^{-1}(3)]}$

$\frac{1}{f'[2]}$

$\boxed{\frac{1}{4}}$

$f^{-1}(3) = \underline{2}$

$f(2) = 3$

$3 = \frac{1}{4}x^3 + x - 1$

$y_1 = \frac{1}{4}x^3 + x - 4$

Find zero

$f'(x) = \frac{3}{4}x^2 + 1 \quad f'(2) = 4$

10. $f(x) = \sqrt{x-4}$ *where* $a = 2$

$f^{-1}(2) = \underline{8}$

$f(8) = 2$

$2 = \sqrt{x-4}$

$4 = x - 4$

$x = 8$

$\frac{1}{f'[f^{-1}(2)]}$

$\frac{1}{f'[8]}$

$\frac{1}{1/4} = \boxed{4}$

$f'(x) = \frac{1}{2}(x-4)^{-1/2}(1)$

$= \frac{1}{2\sqrt{x-4}} = \frac{1}{2\sqrt{4}}$

$\frac{1}{4}$

Use the table to find each derivative.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	**½**	**6**	**-3**	**0**
1	**-1**	**4**	**5**	**-2**

11. $\frac{d}{dx}\left[3f(x) + 2g(x)\right]_{x=-1}$

$3f'(x) + 2g'(x)$

$3f'(-1) + 2g'(-1)$

$3(-3) + 2(0) = \boxed{-9}$

12. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]_{x=1}$

$\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$\frac{g(1)f'(1) - f(1)g'(1)}{[g(1)]^2} = \frac{4(5)-(-1)(-2)}{16}$

$\frac{20-2}{16} = \frac{18}{16} = \boxed{\frac{9}{8}}$

13. $\frac{d}{dx}\left[f(x)g(x)\right]_{x=-1}$

$f(x)g'(x) + g(x)f'(x)$

$f(-1)g'(-1) + g(-1)f'(-1)$

$(\frac{1}{2})(0) + (6)(-3)$

$\boxed{-18}$

14. $\frac{d}{dx}\left[g(f(x))\right]_{x=1}$

$g'[f(x)] \cdot f'(x)$

$g'[f(1)] \cdot f'(1)$

$g'[-1][5] = 0(5) = \boxed{0}$

Calculus : Derivatives

Name ______________________________

Date ________________ Period ______

Find the derivative of each. Simplify appropriately.

15. $y = x^2 \sin x$

$x^2\cos x + \sin x\,[2x]$

$x\,[x\cos x + 2\sin x]$

16. $y = \sec x \tan x$

$\sec x \cdot \sec^2 x + \tan x \sec x \tan x$

$\sec^3 x + \sec x \tan^2 x$

$\sec x\,(\sec^2 x + \tan^2 x)$

17. $y = x^3 \sin x - 5\cos x$

$x^3\cos x + \sin x\,[3x^2] - 5(-\sin x)$

$x^3\cos x + 3x^2\sin x + 5\sin x$

18. $y = 5^x e^x$

$5^x e^x + e^x 5^x \cdot \ln 5(1)$

$5^x e^x\,[1 + \ln 5]$

19. $y = \dfrac{\sec x}{x}$

$\dfrac{x\sec x\tan x - \sec x(1)}{x^2}$

$\dfrac{\sec x\,(x\tan x - 1)}{x^2}$

20. $y = \csc(5 - 2x)$

$-\csc(5-2x)(\cot(5-2x))-2$

$2\csc(5-2x)\cot(5-2x)$

21. $y = (3x^2 - 5x)^3$

$3(3x^2-5x)^2\,[6x-5]$

$3(6x-5)(3x^2-5x)^2$

22. $y = \cos^2(3x - 7)$

$[\cos(3x-7)]^2$

$2[\cos(3x-7)]^1(-\sin(3x-7))(3)$

$-6\cos(3x-7)\sin(3x-7)$

23. $y = \dfrac{1}{\sqrt{2x+1}} = 1(2x+1)^{-1/2}$

$-\frac{1}{2}(2x+1)^{-3/2}(2)$

$\dfrac{-1}{(2x+1)^{3/2}}$

Calculus : Derivatives

Name ______________________

Date ______________ Period ______

24. $y = 7^{\ln x}$

$y' = 7^{\ln x} \cdot \ln 7 \cdot \frac{1}{x}$

$y' = \frac{7^{\ln x} \cdot \ln 7}{x}$

25. $y = \ln\sqrt{x}$

$y = \ln x^{1/2}$

$y = \frac{1}{2}\ln x$

$y' = \frac{1}{2}\left(\frac{1}{x}\right)$

$y' = \frac{1}{2x}$

26. $y = \log_4(\cot x)$

$y' = \frac{1}{\cot x \cdot \ln 4}\left[-\csc^2 x\right]$

$y' = \frac{-\csc^2 x}{\cot x \cdot \ln 4}$

27. $y = \log_3 x^2$

$y = 2\log_3 x$

$y' = 2\,\frac{1}{x \cdot \ln 3} \cdot (1)$

$y' = \frac{2}{x\ln 3}$

28. $y = x^{\sqrt{x}}$

$\ln y = \sqrt{x} \cdot \ln x$

$\frac{1}{y} \cdot \frac{dy}{dx} = \sqrt{x}\left(\frac{1}{x}\right) + \ln x\,\frac{1}{2}x^{-1/2}$

$\frac{dy}{dx} = y\left[\frac{2 \cdot 1}{2\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}\right]$

$\frac{dy}{dx} = x^{\sqrt{x}}\left[\frac{2+\ln x}{2\sqrt{x}}\right]$

29. $y = x^{\frac{1}{x}}$

$\ln y = \frac{1}{x} \cdot \ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\left(\frac{1}{x}\right) + \ln x\frac{d}{dx}[x^{-1}]$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} + \ln x(-x^{-2})$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{\ln x}{x^2}$

$\frac{dy}{dx} = x^{1/x}\left[\frac{1-\ln x}{x^2}\right]$

30. $y = \sin^{-1}(x^3)$

$\frac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2$

$\frac{3x^2}{\sqrt{1-x^6}}$

31. $y = \tan^{-1}\sqrt{x}$

$\frac{1}{(\sqrt{x})^2+1}\;\frac{1}{2}x^{-1/2}$

$\frac{1}{2\sqrt{x}\,[x+1]}$

32. $y = \cos^{-1}\frac{1}{x}$

$\frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot \frac{d}{dx}[x^{-1}]$

$\frac{-1}{\sqrt{1-\frac{1}{x^2}}}\,(-1)x^{-2}$

$\frac{1}{x^2\sqrt{1-1/x^2}}$

$\frac{1}{x^2\sqrt{\frac{x^2-1}{x^2}}} = \frac{1}{x\sqrt{x^2-1}}$