AP Calculus
Definite Integrals and Rate of Change
Name $\qquad$ 1-4: Evaluate each integral.

1. $\int_{1}^{4} \frac{\sqrt{y}-y}{y^{2}} d y$
2. $\int_{0}^{1}\left(5 x-5^{x}\right) d x$
3. $\int_{1}^{64} \frac{1+\sqrt[3]{x}}{\sqrt{x}} d x$
4. $\int_{0}^{\frac{\sqrt{3}}{2}} \frac{d r}{\sqrt{1-r^{2}}}$
5. If $w^{\prime}(t)$ is the rate of growth of a child in pounds per year, what does $\int_{5}^{10} w^{\prime}(t) d t$ represent?
6. If oil leaks from a tank at a rate of $\mathrm{r}(\mathrm{t})$ gallons per minute at time t , what does $\int_{0}^{120} r(t) d t$ represent?
7. A honeybee population starts with 100 bees and increases at a rate of $n^{\prime}(t)$ bees per week. What does $100+\int_{0}^{15} n^{\prime}(t) d t$ represent?

8-9: The velocity function (in meters per second) is given for a particle moving along a line. Find a.) the displacement and b.) the distance traveled by the particle during the given time
8. $v(t)=3 t-5, \quad 0 \leq t \leq 3$
9. $v(t)=t^{2}-2 t-8, \quad 1 \leq t \leq 6$

10-11: The acceleration function (in $\mathrm{m} / \mathrm{s}^{2}$ ) and the initial velocity are given for a particle moving along a line. Find a.) the velocity at time $\dagger$ and b.) the distance traveled during the given time
10. $a(t)=t+4$,
$v(0)=5, \quad 0 \leq t \leq 10$
11. $a(t)=2 t+3, \quad v(0)=-4, \quad 0 \leq t \leq 3$
12. The linear density of a rod of length 4 m is given by $p(x)=9+2 \sqrt{x}$ measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod.
13. Water flows from the bottom of a storage tank at a rate of $r(t)=200-4 t$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.
14. The velocity of a car was read from its speedometer at 10 -second intervals and recorded in the table. Use the Midpoint Rule to estimate the distance traveled by the car.

| $t(s)$ | $v(\mathrm{mi} / \mathrm{h})$ | $\mathrm{t}(\mathrm{s})$ | $\mathrm{v}(\mathrm{mi} / \mathrm{h})$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 60 | 56 |
| 10 | 38 | 70 | 53 |
| 20 | 52 | 80 | 50 |
| 30 | 58 | 90 | 47 |
| 40 | 55 | 100 | 45 |
| 50 | 51 |  |  |

15. Water flows into and out of a storage tank. A graph of the rate of change $r(t)$ of the volume of water in the tank, in liters per day, is shown. If the amount of water in the tank at time $t=0$ is $25,000 \mathrm{~L}$, use the Midpoint Rule of estimate the amount of water in the tank four days later.


## Answers:

1.) $\quad 1-\ln (4)$
2.) $\frac{5}{2}-\frac{4}{\ln (5)}$
3.) $\frac{256}{5}$
4.) $\frac{\pi}{3}$
5.) The total pounds a child grew from age 5 to age 10.
6.) The total gallons of oil that leaked over the 120 minutes.
7.) The total number of bees after 15 weeks.
8.) Displacement $=-\frac{3}{2}$ Distance $=6.833$
10.) $v(t)=\frac{1}{2} t^{2}+4 t+5$ Distance $=\int_{0}^{10}|v(t)|=416.666$
12.) 46.666 kg
13.) 1800 liters
9.) Displacement $=-\frac{10}{3}$ Distance $=32.666$
11.) $v(t)=t^{2}+3 t-4$ Distance $=\int_{0}^{3}|v(t)|=14.833$
14.) $\quad 1.372$ miles $\quad$ 15.) $\quad 28,250$ liters

