AP Calculus
Differential Equations
1-7: Solve the differential equation.

Name_____
Application of Integration Day 8

1. $\frac{dy}{dx} = xy^2$

2. $\frac{dy}{dx} = xe^{-y}$

$dy = xe^{-y}dx$

$\frac{1}{e^{-y}}dy = xdx$

$\int e^y dy = \int x dx$

$e^y = \frac{1}{2}x^2 + C$

$\ln e^y = \ln(\frac{1}{2}x^2 + C)$

$y = \ln(\frac{1}{2}x^2 + C)$

3. $xy^2y' = x+1$

4. $(y^2 + xy^2)y' = 1$

$y^2(1+x)\frac{dy}{dx} = 1$

$y^2(1+x)dy = dx$

$\int y^2 dy = \int \frac{1}{1+x} dx$

$u = 1+x \quad du = dx$

$\frac{y^3}{3} = \int \frac{1}{u} du$

$\frac{1}{3}y^3 = \ln|1+x| + C$

$y^3 = 3\ln|1+x| + C$

$y = \sqrt[3]{3\ln|1+x| + C}$

5. $(y + \sin y)y' = x + x^3$

6. $\frac{dp}{dt} = t^2p - p + t^2 - 1$

$\frac{dP}{dt} = P(t^2-1) + t^2 - 1$

$\frac{dP}{dt} = P(t^2-1) + 1(t^2-1)$

$\frac{dP}{dt} = (P+1)(t^2-1)$

$\int \frac{1}{P+1} dP = \int t^2 - 1 dt$

$u = P+1 \quad du = dP$

$\int \frac{1}{u} du = \frac{t^3}{3} - t + C$

$\ln|P+1| = \frac{t^3}{3} - t + C$

$e \qquad e^{\frac{t^3}{3} - t}$

$P+1 = Ce^{\frac{1}{3}t^3 - t}$

$P = Ce^{\frac{1}{3}t^3 - t} - 1$

7. $\frac{dz}{dt} + e^{t+z} = 0$

8-11: Find the solution of the differential equation that satisfies the given initial condition.

8. $\frac{dy}{dx}=\frac{x}{y}$, $y(0)=-3$ (x, y)

$\int y\,dy=\int x\,dx$

$\frac{y^2}{2}=\frac{x^2}{2}+C$

$\frac{9}{2}=\frac{0}{2}+C$

$C=\frac{9}{2}$

$\frac{y^2}{2}=\frac{x^2}{2}+\frac{9}{2}$

$y^2=x^2+9$

$y=\pm\sqrt{x^2+9}$

$-3=\pm\sqrt{0^2+9}$

$\boxed{y=-\sqrt{x^2+9}}$

9. $\frac{dy}{dx}=\frac{\ln x}{xy}$, $y(1)=2$

10. $\frac{du}{dt}=\frac{2t+\sec^2 t}{2u}$, $u(0)=-5$ (t, u)

$\int 2u\,du=\int 2t+\sec^2 t\,dt$

$\frac{2u^2}{2}=\frac{2t^2}{2}+\tan t+C$

$u^2=t^2+\tan t+C$

$(-5)^2=0^2+\tan(0)+C$

$C=25$

$u^2=t^2+\tan t+25$

$u=\pm\sqrt{t^2+\tan t+25}$

$-5=\pm\sqrt{0^2+\tan(0)+25}$

$\boxed{u=-\sqrt{t^2+\tan t+25}}$

11. $\frac{dP}{dt}=\sqrt{Pt}$, $P(1)=2$

12. Find an equation of the curve that passes through the point $(0,1)$ and whose slope at (x,y) is xy.

$\frac{dy}{dx} = xy$

a derivative is a slope.

$\frac{1}{y}dy = x\,dx$

$\int \frac{1}{y}dy = \int x\,dx$

$\ln|y| = \frac{1}{2}x^2 + C$

$\ln|1| = \frac{1}{2}(0)^2 + C$

$C = 0$

$e^{\ln|y|} = e^{\frac{1}{2}x^2}$

$|y| = e^{\frac{1}{2}x^2}$

$y = \pm e^{\frac{1}{2}x^2}$

$y = e^{\frac{1}{2}x^2}$

13. A.) Solve the differential equation $y' = 2x\sqrt{1-y^2}$.

B.) Solve the initial-value problem $y' = 2x\sqrt{1-y^2}$, $y(0) = 0$, and graph the solution.

C.) Does the initial-value problem $y' = 2x\sqrt{1-y^2}$, $y(0) = 2$, have a solution? Explain.

Review: ☺ Must show your work to get credit ☺ **NON-CALCULATOR**

14. A Hillgrove student, 5 feet tall, is walking away from a lamppost which is 12 feet tall. She walks away at a constant rate of 2 feet per second and notices that, as she moves away from the lamppost, the length of her shadow in increasing. How fast is the length of her shadow increasing in feet per second when she is 20 feet from the post.

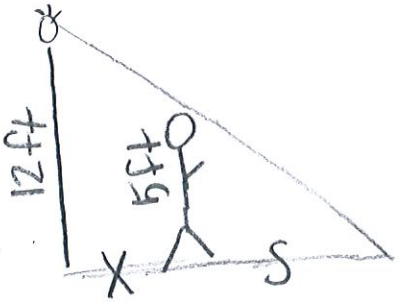
a.) $\frac{7}{10}\frac{ft}{sec}$

(b) $\frac{10}{7}\frac{ft}{sec}$

fc) $2\frac{ft}{sec}$

d) $\frac{34}{7}\frac{ft}{sec}$

e) $\frac{27}{10}\frac{ft}{sec}$



Eqn: $\frac{12}{x+s}=\frac{5}{s}$

Know: $\frac{dx}{dt}=2\frac{ft}{sec}$

$12s=5x+5s$

$7s=5x$

Find: $\frac{ds}{dt}=$

Dee: $7\frac{ds}{dt}=5\frac{dx}{dt}$

When: $x=20ft$

Sub: $\frac{ds}{dt}=\frac{5}{7}\cdot\frac{2ft}{sec}=\frac{10}{7}\frac{ft}{sec}$

15. Find $\int\frac{x^3-x^2+1}{x}dx$.

a.) $2x-1+C$

b) $\frac{x^3}{3}-\frac{x^2}{2}+\ln x+C$

c) $\frac{x^3}{3}-\frac{x^2}{2}-\frac{1}{x^2}+C$

d) $\frac{x^3}{3}-\frac{x^2}{2}+\frac{1}{x^2}+C$

(e) $\frac{x^3}{3}-\frac{x^2}{2}+\ln|x|+C$

$\int x^2-x+\frac{1}{x}dx$

$\frac{x^3}{3}-\frac{x^2}{2}+\ln|x|+C$

Answers:

1.	$y=\frac{-1}{\frac{1}{2}x^2+C}$	2.	$y=\ln\left(\frac{1}{2}x^2+C\right)$	3.	$y=\sqrt[3]{3x+3\ln\lvert x\rvert+C}$
4.	$y=\sqrt[3]{3\ln\lvert 1+x\rvert+C}$	5.	$2y^2-4\cos y=2x^2+x^4+C$	6.	$P=Ce^{\frac{1}{3}t^3+t}-1$
7.	$z=-\ln\left(e^t+C\right)$	8.	$y=-\sqrt{x^2+9}$	9.	$y=\sqrt{\ln\lvert x\rvert+4}$
10.	$u=-\sqrt{t^2+\tan t+25}$	11.	$P=\left(\frac{1}{3}t^{\frac{3}{2}}+\sqrt{2}-\frac{1}{3}\right)^2$	12.	$y=e^{\frac{1}{2}x^2}$
13.	a.) $y=\sin\left(x^2+C\right)$	b.)	$y=\sin\left(x^2\right)$	c.)	No! $\sin(x)\neq 2$
14.	B	15.	E		