

AP Calculus

Derivatives of Logarithmic Functions

1-10: Differentiate the function.

1. $f(x) = x \ln x - x$

2. $f(x) = \sin(\ln x)$

$$f'(x) = \cos(\ln x) \cdot \frac{d}{dx} [\ln x]$$

$$f'(x) = \cos(\ln x) \left(\frac{1}{x} \right)$$

$$f'(x) = \frac{\cos(\ln x)}{x}$$

3. $f(x) = \ln(\sin^2 x)$

4. $f(x) = \ln \frac{1}{x} = \ln(x^{-1})$

$$f'(x) = \frac{1}{x^{-1}} \frac{d}{dx} [x^{-1}] = \frac{1}{x^{-1}} (-x^{-2})$$

$$f'(x) = \frac{-x^{-2}}{x^{-1}} = \frac{-x^{-1}}{x^2} = \frac{-1}{x}$$

5. $y = \frac{1}{\ln x}$

6. $f(x) = \log_{10}(x^3 + 1)$

$$f'(x) = \frac{1}{(x^3 + 1) \ln(10)} [3x^2]$$

$$f'(x) = \frac{3x^2}{(x^3 + 1) \ln(10)}$$

7. $f(x) = \log_5(xe^x)$

8. $f(x) = \sin x \ln(5x)$

$$f'(x) = \sin x \cdot \frac{1}{5x} \cdot 5 + \ln(5x) \cos x$$

$$= \frac{\sin x}{x} + \ln(5x) \cos x$$

9. $g(x) = \ln(x\sqrt{x^2 - 1})$

10. $g(r) = r^2 \ln(2r + 1)$

$$g'(r) = r^2 \left(\frac{1}{2r+1} \right) (2) + \ln(2r+1) [2r]$$

$$g'(r) = \frac{2r^2}{2r+1} + 2r \ln(2r+1)$$

Derivatives of Logarithmic Functions

D(1) Day 8

11-12: Find an equation of the tangent line to the curve at the given point.

11. $y = \ln(x^2 - 3x + 1)$, (3,0) Point $y(3) = 0$

$$y' = \frac{1}{x^2 - 3x + 1} \cdot \frac{d}{dx}[x^2 - 3x + 1] \quad \text{slope } y'(3) = 3$$

$$y' = \frac{1}{x^2 - 3x + 1} \cdot (2x - 3) \quad \boxed{y - 0 = 3(x - 3)}$$

$$y'(x) = \frac{2x - 3}{x^2 - 3x + 1} \quad y'(3) = \frac{2(3) - 3}{3^2 - 3(3) + 1} = \frac{3}{1} = 3$$

12. $y = x^2 \ln x$, (1,0)

13-18: Use logarithmic differentiation to find the derivative of the function.

13. $y = (x+2)^2 (x^4 + 4)^4$

14. $y = \sqrt{\frac{x-1}{x^4+1}}$

15. $y = x^x$

16. $y = x^{\cos x}$

17. $y = x^{\sin x}$

18. $y = \sqrt{x}^x$