Name **BC Calculus – Integrals**

Evaluate the integral.

1. $\int \frac{6x+1}{5x+2}dx$

$$5x+2\,\overline{)\,6x+1}\quad \frac{6}{5}+\frac{-\frac{7}{5}}{5x+2}$$

$$\ominus\, 6x+\frac{12}{5}$$

$$-\frac{7}{5}$$

$$\int \frac{6}{5}+\frac{-\frac{7}{5}}{5x+2}\,dx$$

$$\boxed{\frac{6}{5}x-\frac{7}{25}\ln|5x+2|+C}$$

2. $\int_1^{\infty} x^{-\frac{3}{2}}dx$

$$\lim_{b\to\infty}\int_1^b x^{-3/2}\,dx$$

$$=\lim_{b\to\infty} -2x^{-1/2}\Big|_1^b$$

$$=\lim_{b\to\infty}\frac{-2}{\sqrt{x}}\Big|_1^b=\lim_{b\to\infty}\overset{\nearrow 0}{\frac{-2}{\sqrt{b}}}+\frac{2}{\sqrt{1}}\cdot\boxed{2}$$

3. $\int x^4\ln x\,dx$

$u=\ln x \qquad dv=x^4dx$

$du=\frac{1}{x}dx \qquad v=\frac{x^5}{5}$

$$=\frac{x^5}{5}\ln x-\int\frac{x^5}{5}\cdot\frac{1}{x}dx$$

$$=\frac{x^5}{5}\ln x-\int\frac{x^4}{5}dx$$

$$=\boxed{\frac{x^5}{5}\ln x-\frac{x^5}{25}+C}$$

4. $\int \frac{1}{x^2-4x-12}dx$

$$\frac{A}{(x-6)}+\frac{B}{(x+2)}$$

$A(x+2)+B(x-6)=1$

$x=-2:\ -8B=1$

$B=-\frac{1}{8}$

$x=6:\ 8A=1$

$A=\frac{1}{8}$

$$\int\frac{\frac{1}{8}}{x-6}+\frac{-\frac{1}{8}}{x+2}\,dx$$

$$\boxed{\frac{1}{8}\ln|x-6|-\frac{1}{8}\ln|x+2|+C}$$

5. $\int x\sin x\,dx$

$u=x \qquad dv=\sin x\,dx$

$du=dx \qquad v=-\cos x$

$$-x\cos x-\int -\cos x\,dx$$

$$\boxed{-x\cos x+\sin x+C}$$

6. $\int_0^2 \frac{1}{(x-1)^2}dx$

$$\int\frac{1}{(x-1)^2}=\int(x-1)^{-2}dx$$

$$=-(x-1)^{-1}=\frac{-1}{x-1}$$

$$\lim_{b\to1^-}\int_0^b\frac{1}{(x-1)^2}dx+\lim_{a\to1^+}\int_a^2\frac{1}{(x-1)^2}dx$$

$$\lim_{b\to1^-}\frac{-1}{x-1}\Big|_0^b+\lim_{a\to1^+}\frac{-1}{x-1}\Big|_a^2\quad\boxed{\infty}\ \boxed{\text{diverges}}$$

$$\lim_{b\to1^-}\frac{-1}{b-1}+\frac{1}{0-1}+\lim_{a\to1^+}\frac{-1}{2-1}+\frac{1}{a-1}$$

$$=\underset{\infty}{\frac{-1}{.999-1}}-\frac{1}{1}-\frac{1}{1}+\underset{\infty}{\frac{1}{1.9}}$$

7. $\int\frac{dx}{x^2+x}$ $=\int\frac{dx}{x(x+1)}$

$$\frac{A}{x}+\frac{B}{x+1}=\frac{1}{x(x+1)}$$

$A(x+1)+Bx=1$

$x=-1:\ B=-1$

$x=0\ \ A=1$

$$\int\frac{1}{x}+\frac{-1}{x+1}$$

$$\boxed{\ln|x|-\ln|x+1|+C}$$

8. $\int\frac{x^2+2x}{x+2}dx$

$$\int\frac{x\cancel{(x+2)}}{\cancel{x+2}}\,dx$$

$$\int x\,dx=\boxed{\frac{x^2}{2}+C}$$

9. $\int \frac{dx}{4+x^2}$ $\frac{1}{4}\int \frac{dx}{1+\frac{x^2}{4}}$

$\frac{1}{4}\int \frac{dx}{1+(\frac{x}{2})^2}$ $u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2du = dx$

$\frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1}u + C$

$= \boxed{\frac{1}{2}\tan^{-1}(\frac{x}{2}) + C}$

10. $\int x^2 e^{-3x}\,dx$

	u	dv
$+$	x^2	e^{-3x}
$-$	$2x$	$\frac{e^{-3x}}{-3}$
$+$	2	$\frac{e^{-3x}}{9}$
$-$	0	$\frac{e^{-3x}}{-27}$

$\boxed{\frac{x^2e^{-3x}}{-3} - \frac{2xe^{-3x}}{9} - \frac{2e^{-3x}}{27} + C}$

11. $\int \frac{\ln(\ln x)}{x}dx$ $u = \ln x$

$du = \frac{1}{x}dx$

$\int \ln(u)\,du$

$u\ln u - u + C$

$\boxed{\ln x(\ln(\ln x)) - \ln x + C}$

12. $\int \cos^3 x\,dx$

$= \int \cos^2 x \cos x\,dx$ $\cos^2 x = 1 - \sin^2 x$

$= \int (1-\sin^2 x)\cos x\,dx$ $u = \sin x$

$du = \cos x\,dx$

$= \int (1-u^2)\,du$

$= u - \frac{u^3}{3} + C = \boxed{\sin x - \frac{\sin^3 x}{3} + C}$

13. $\int \cos^2 x \sin^2 x\,dx$ $\cos^2 x = \frac{1}{2}(1+\cos(2x))$

$\sin^2 x = \frac{1}{2}(1-\cos(2x))$

$\int \frac{1}{2}(1+\cos(2x)) \cdot \frac{1}{2}(1-\cos(2x))\,dx$

$\frac{1}{4}\int (1+\cos(2x))(1-\cos(2x))\,dx$

$\frac{1}{4}\int 1-\cos^2(2x)\,dx$ $\cos^2(2x) = \frac{1}{2}(1+\cos(4x))$

$\frac{1}{4}\int 1 - \frac{1}{2}(1+\cos(4x))\,dx$

$\frac{1}{4}\int 1 - \frac{1}{2} - \frac{1}{2}\cos(4x)\,dx$

$\frac{1}{4}\int \frac{1}{2} - \frac{1}{2}\cos(4x)\,dx$

$\frac{1}{4}\left[\frac{1}{2}x - \frac{\sin(4x)}{8}\right] + C = \boxed{\frac{1}{8}x - \frac{\sin(4x)}{32} + C}$

14. $\int_{-\infty}^{0} e^{3x}\,dx$

$\lim_{a\to-\infty} \int_a^0 e^{3x}\,dx$

$= \lim_{a\to-\infty} \frac{e^{3x}}{3}\Big|_a^0$

$= \lim_{a\to-\infty} \frac{e^0}{3} - \frac{e^{3a}}{3}$

$= \lim_{a\to-\infty} \frac{1}{3} - 0 = \boxed{\frac{1}{3}}$