Notes: Alternating Series, Absolute Convergence, & Conditional Convergence

Infinite Series Day 8

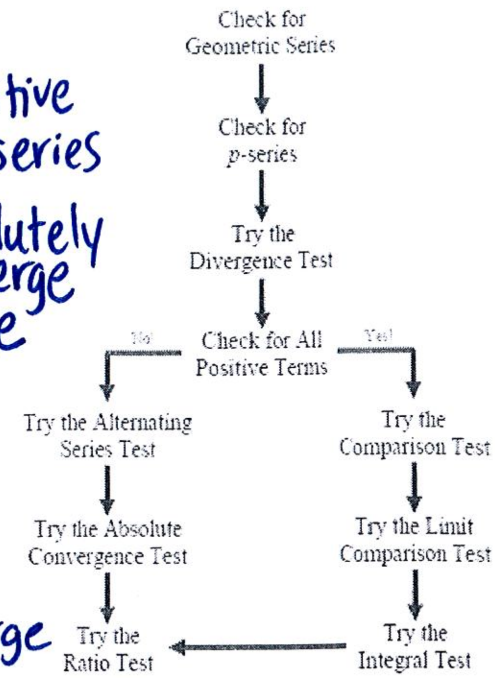
Absolute Convergence Test:

The series $\sum a_n$ is *absolutely convergent* if $\sum |a_n|$ is convergent. If $\sum a_n$ is absolutely convergent, then $\sum a_n$ is convergent

If $\sum_{n=1}^{\infty}$ All positive term series

Then $\boxed{1}$ Absolutely converge

$\boxed{2}$ Diverge



If $\sum_{n=1}^{\infty}$ Alternating series

Conditional Convergence:

$\sum a_n$ is *conditionally convergent* if $\sum a_n$ converges, but $\sum |a_n|$ diverges.

$\boxed{1}$ Absolutely Converge

$\boxed{2}$ Conditionally Converge

$\boxed{3}$ diverge

Determine Whether the series converges absolutely or conditionally, or not at all = diverge

Example One:

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

Alternating

Look at Positive Series First

$\sum_{n=1}^{\infty} \frac{1}{n}$

diverges by p-series $p = 1 \leq 1$.

Look at Alternating

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

Alternating & $\lim_{n \to \infty} \frac{1}{n} = 0$

$\therefore$ converges by Alternating

Converges conditionally

Example Two:

$\sum_{n=1}^{\infty} \frac{\cos(2n)}{n^4}$

Alternating

Look at Positive Series First

$\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges by p-series $p = 4 > 1$.

Converges absolutely

If the positive series converges You do not have to check. The Alternating will too.

Example Three:

$$\sum_{n=0}^{\infty} (-1)^{n-1} \frac{2^n - 5^n}{4^n} = \sum (-1)^{n-1} \left(\frac{2}{4}\right)^n - \sum (-1)^{n-1} \left(\frac{5}{4}\right)^n$$

Alternating

No need to look at positive series & alternating geometric = $|R|$ which means same answer for positive or negative!

$|R| = \frac{2}{4} = \frac{1}{2} < 1$ converges by geometric

$|R| = \frac{5}{4} > 1$ diverges by geometric

diverges