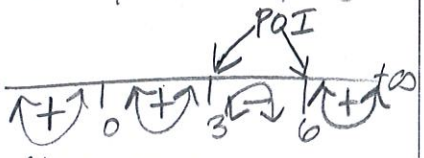
Review: Curve Sketching $y_1 = x^2 \cdot \cos(x^2)$ Name_____

1. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x −coordinates of the points of inflection of the graph of f? $x^2=0$ $x-3=0$ $x-6=0$ $x=0$ $x=3$ $x=6$

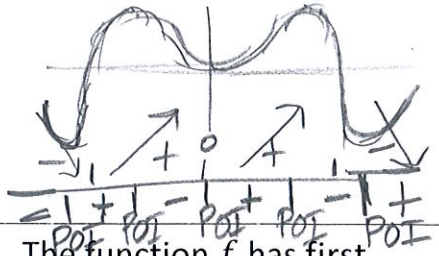
(A) 0 only
(B) 3 only
(C) 0 and 6 only
(D) 3 and 6 only
(E) 0, 3, and 6



$f''(-1) = (+)(-)(-) = +$ $f''(1) = (+)(-)(-) = +$
$f''(4) = (+)(+)(-) = -$ $f''(7) = (+)(+)(+) = +$

Calculator 2. The derivative of the function f is given by $f'(x) = x^2\cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2,2)$? $y_1 = x\cos(x^2)$ $(-2,2)$

(A) One
(B) Two
(C) Three
(D) Four
(E) Five



3. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when $f'(x) = 2xe^x + e^x(2)$

(A) $x < -2$
(B) $x > -2$
(C) $x < -1$
(D) $x > -1$
(E) $x < 0$

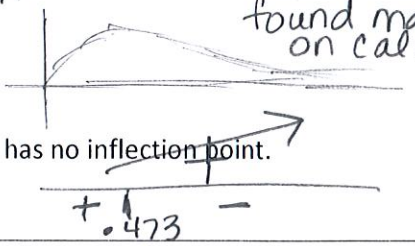
$f'(x) = 2e^x(x+1)$
$f''(x) = 2e^x(1) + (x+1)2e^x$
$f''(x) = 2e^x(1+x+1)$
$f''(x) = 2e^x(x+2)$
$2e^x = 0$ garbage $x+2=0$ $x=-2$

$f''(0) = (+)(+)$
$f''(-3) = (+)(-)(-)$

Calculator 4. The function f has first derivative given by $f'(x) = \frac{\sqrt{x}}{1+x+x^3}$. What is the x −coordiante of the inflection point of the graph of f? $y_1 = (\sqrt{x}) \div (1+x+x^3)$ $(-1,2)$

(A) 1.008
(B) 0.473
(C) 0
(D) −0.278
(E) The graph of f has no inflection point.

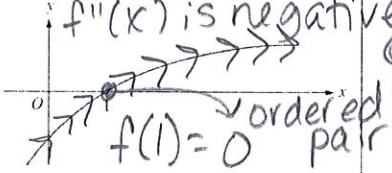
found max on calc



5. The graph of a twice-differentiable function f is shown in the figure. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

f is increasing
$f'(x)$ = positive
f is concave down
$f''(x)$ is negative



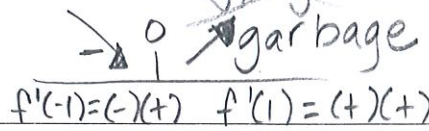
ordered pair
$f(1) = 0$

$f''(1) < f(1) < f'(1)$
neg < 0 < pos

6. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$
(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
(C) $(0, \infty)$
(D) $(-\infty, 0)$
(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

$f'(x) = 4x^3 + 2x$
$0 = 2x(x^2+1)$
$2x = 0$ $x^2+1=0$
$\sqrt{x^2} = \sqrt{-1}$ garbage



$f'(-1) = (-)(+)$ $f'(1) = (+)(+) = +$

7. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.
(B) $x < -1$ and $x > 3$
(C) $-3 < x < 1$
(D) $-1 < x < 3$
(E) All values of x

$f'(-2) = -$ $f'(4) = -$
$f'(0) = +$

$f'(x) = (x^2-3)e^{-x}(-1) + e^{-x}(2x)$
$f'(x) = e^{-x}(-x^2+3+2x)$
$f'(x) = e^{-x}(-x^2+2x+3)$
$e^{-x} = 0$ garbage
$-x^2+2x+3=0$
$x^2-2x-3=0$ $(x-3)(x+1)$ $x=3, x=-1$

Calculator 8. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) −1.58
(B) −1.63
(C) −1.67
(D) −1.89
(E) −2.33

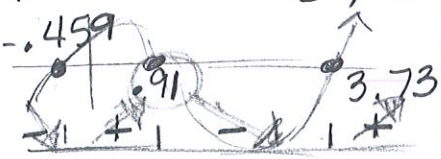
$y' = 3x^2 + 12x + 7 - 2(-\sin x)$
$y' = 3x^2 + 12x + 7 + 2\sin x$
$y'' = 6x + 12 + 2\cos x$
$0 = 6x + 12 + 2\cos x$
$y_1 = 6x + 12 + 2\cos x$ $[-3, -1]$
$x = -1.8941$

$6x-12=0$ $x^5=0$ $+6-\frac{1}{2}+$
$6x=12$ $x=0$
$x=2$

Calculator 9. If the derivative of f is given by $f'(x)=e^x-3x^2$, at which of the following values of x does f have a relative maximum value?

$Y_1=e^x-3x^2$ $[-1,4]$

(A) -0.46
(B) 0.20
(C) 0.91
(D) 0.95
(E) 3.73



10. At what value of x does the graph of $y=\frac{1}{x^2}-\frac{1}{x^3}$ have a point of inflection?

Not in domain of original

(A) 0
(B) 1
(C) 2
(D) 3
(E) At no value of x

$y=x^{-2}-x^{-3}$
$y'=-2x^{-3}+3x^{-4}$
$y''=6x^{-4}-12x^{-5}$
$=\frac{6}{x^4}-\frac{12}{x^5}=\frac{6x-12}{x^5}$

11. How many critical points does the function $f(x)=(x+2)^5(x-3)^4$ have?

(A) *One*
(B) *Two*
(C) *Three*
(D) *Five*
(E) *Nine*

$f'(x)=(x+2)^5(4)(x-3)^3(1)+(x-3)^4 5(x+2)^4(1)$
$f'(x)=(x+2)^4(x-3)^3[4(x+2)+5(x-3)]$
$f'(x)=(x+2)^4(x-3)^3[4x+8+5x-15]$
$0=(x+2)^4(x-3)^3(9x-7)$

12. The graph of $y=\frac{-5}{x-2}$ is concave downward for all values of x such that

(A) $x<0$
(B) $x<2$
(C) $x<5$
(D) $x>0$
(E) $x>2$

$y'=\frac{(x-2)(0)-(-5)(1)}{(x-2)^2}$
$y'=\frac{5}{(x-2)^2}$
$y''=\frac{(x-2)^2(0)-5(2)(x-2)^1(1)}{(x-2)^4}$
$\frac{-10(x-2)}{(x-2)^4}=\frac{10}{-(x-2)^3}$
$+\ 1\ 2\ -$

13. Let f be a polynomial function with degree greater than 2. If $a\neq b$ and $f(a)=f(b)=1$, which of the following must be true for at least one value of x between a and b ?

I. $f(x)=0$
II. $f'(x)=0$
III. $f''(x)=0$

Rolle's Thm

(A) *None*
(B) *I only*
(C) *II only*
(D) *I and II only*
(E) *I, II, and III*

14. The absolute maximum value of $f(x)=x^3-3x^2+12$ on the closed interval $[-2,4]$ occurs at $x=$

(A) 4
(B) 2
(C) 1
(D) 0
(E) -2

$f'(x)=3x^2-6x$
$0=3x(x-2)$
$x=0$ $x=2$
$f(-2)=-8$
$f(0)=12$
$f(2)=8$
$f(4)=28$ (Absolute max)

15. The function defined by $f(x)=x^3-3x^2$ for all real numbers x has a relative maximum at $x=$

(A) -2
(B) 0
(C) 1
(D) 2
(E) 4

$f'(x)=3x^2-6x$
$0=3x(x-2)$
$x=0$ $x=2$
$f'(-1)=(-)(-)=+$
$f'(1)=+(-)$
$f'(3)=(+)(+)$
$+\ 0\ -\ 2\ +$

16. If $f(x)=\frac{\ln x}{x}$, for all $x>0$, which of the following is true?

$f'(x)=\frac{x(\frac{1}{x})-\ln x(1)}{x^2}$
$f'(x)=\frac{1-\ln x}{x^2}$
$9+\ e\ -$
$1-\ln x=0$ $\ln x=1$ $x=e$ $x=0$

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

Calculator 17. The graph of $y=5x^4-x^5$ has a point of inflection at

(A) (0,0) *only*
(B) (3,162) *only*
(C) (4,256) *only*
(D) (0,0) *and* (3,162)
(E) (0,0) *and* (4,256)

$y'=20x^3-5x^4$
$y''=60x^2-20x^3$
$0=20x^2(3-x)$
$x=0$ $x=3$
$+\ 0\ +\ 3\ -$
$f''(-1)=(+)(+)$
$f''(1)=(1)(+)$
$f''(4)=(+)(-)$

18. At $x=0$, which of the following is true of the function f defined by $f(x)=x^2+e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

$f'(x)=2x+e^{-2x}(-2)$
$f'(x)=2x-2e^{-2x}$
$f'(0)=2(0)-2e^0=-2=$ neg.